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DYNAMICS OF LIQUID-FILLED PROJECTILES

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This report presents a collection of analytic models which treat various aspects of the dynamics of liquid-filled, spin-stabilized projectiles. Several numerical examples are given applicable to eight-inch ammunition. Although idealized, these examples may provide understanding of the behavior of real systems such as the XM736 binary round.

Chapter 1 examines the change in inertial characteristics with a change in liquid configuration. Chapter 2 treats the

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dynamics of spinup of the liquid fill. Also considered are the range and deflection sensitivity to configuration change, and the question of "ballistic similitude" with a comparable solid-filled projectile. The distribution of energy in the liquid and an approximation for the frequency of a fundamental vibratory mode of the liquid are addressed in Chapter 3. The frequency of vibration is compared with precessional and nutational frequencies of the projectile during flight to assess the likelihood of stability problems.

Insofar as the approximations made here are applicable, one can conclude that the XM736 projectile is a reasonable ballistic match to the M509. Difference in mean point of impact can be corrected by small adjustments in aiming. The XM736 is judged to be ballistically stable but will likely have a somewhat larger ballistic dispersion than either the M509 or M106 projectiles.

PREFACE

The author gratefully acknowledges the assistance of his wife, Emily, in typing the manuscript.

The author is indebted to Bill D'Amico of the Ballistics Research Laboratory for reviewing a draft of this report and only regrets that time and project priorities did not permit him to pursue Bill's many interesting suggestions.

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INTRODUCTION

The initial objectives of this study were quite simple, namely, to analytically determine whether a liquid-filled projectile having a partial fill similar to the XM736, eight-inch binary projectile would be an adequate ballistic match to a comparable solid projectile, the M509. This study is an effort to respond to a portion of a study request of the XM736 projectile initiated by the U.S. Army Materiel Command. During our investigation the objectives and techniques were expanded. To a great extent the study assumed a methodological orientation, with the methods illustrated by simple examples.

In an effort to keep the report unclassified some specificity and fidelity to actual developmental configurations had to be sacrificed. For example, the effect of change in liquid position on inertial properties was examined via a hypothetical, simplified, liquid-filled, projectile configuration. Hopefully, this idealization does not preclude application to real systems. All numerical examples given here were chosen from the eight-inch family of ammunition. However, the methods used are considered to be applicable to other spin-stabilized, liquid-filled projectiles.

Insofar as the initial study objectives are concerned, one can conclude that the XM736 projectile is a reasonable ballistic match to the M509. Difference in mean point of impact can be corrected by small adjustments in aiming. The XM736 is judged to be ballistically stable but will likely have a somewhat larger ballistic dispersion than either the M509 or M106 projectiles.

These conclusions certainly can be challenged on the grounds that the methods and/or data inputs are inapplicable

to the XM736 projectile. Admittedly, the phenomena occuring during spinup and mixing of the chemical reagents in that system are not treated adequately in this study. In fact, limitations of the study are recognized and stated explicitly in the caveats below.

Although lacking physical rigor at several points, this report is offered in the hope that some of the methods and insights may be of interest and that some of the physical shortcomings may provoke further inquiry. The author suggests that a careful, finite-element approach to the analysis of mixing and spinup, considering pertinent physical phenomena, may be a useful avenue of approach.

Caveats

The following, numbered caveats are provided to identify restrictions in the scope and depth of the study:

1. Treatment of phenomena occuring during mixing of chemical reagents (for binary systems) is highly idealized.
   a. Constant free volume within the shell cavity was assumed although it is known that prior to mixing the liquid ingredients in the XM736 occupy a smaller volume than that assumed, and that after mixing and reacting chemically, essentially the entire cavity is occupied by reaction products.
   b. The value of kinematic viscosity was treated parametrically at two (constant) levels -- 10 and 1 centistoke. It is estimated from laboratory tests that the viscosity of the cold reactants is greater than ten centistokes whereas the hot reaction products have a viscosity of less than one centistoke.
2. Only certain kinds of liquid resonances were treated. It is known that pressure waves propagating in a liquid

confined within a cavity have certain characteristic- or eigen-frequencies. Although not treated in this report, a discussion of these resonant frequencies is found in AMCP-706-165, Liquid-Filled Projectile Design. The discussion of liquid resonances in this report is limited to the whole-body or sloshing motion of the liquid. Obviously, this discussion applies only to partially-filled shells.

3. An academic or idealized mathematical treatment of liquid spinup is contained herein. This development strictly applies only when liquid flow is laminar and, then, only for cavities which are quite long relative to their diameter. The diffusion of angular momentum within the liquid due to turbulence, generated either by a chemical reaction or by vorticity cells induced by the boundary conditions at the ends of the liquid cylinder, were not considered in this study. For most liquid-filled projectiles, the rapid achievement of homogenous angular velocity is due mainly to the turbulent condition of the liquid during spinup which, effectively, creates a large apparent viscosity.

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CHAPTER 1

CHANGE IN INERTIAL CHARACTERISTICS OF A LIQUID-FILLED PROJECTILE DURING FLIGHT

Inertial Characteristics in Two Limiting Configurations

Consider the following idealized model of a liquid-filled projectile during launch. Because of the large axial acceleration and small average angular velocity of the liquid, the liquid will be forced to the rear of the shell cavity so that its free surface will approximate a flat circular diaphragm. See Example 1 below. This liquid configuration, A, is shown below in Figure 1.1.

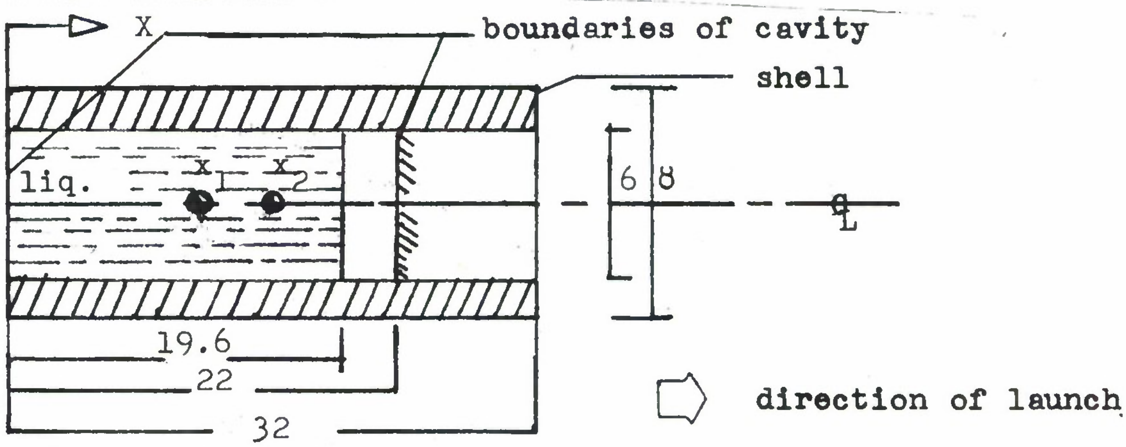

Figure 1.1. Configuration A

The solid shell body is approximated as a cylindrical sleeve 32 in. long, with a 1 in. wall thickness. Massless boundaries confine the liquid to a cylindrical cavity 22 in. long, having a 6 in. diameter. The liquid fills only 90% of the available volume. During launch the height of the column of liquid is 19.6 in. in the axial or X-direction. The average density of the solid shell body is assumed to be 0.2558 lb/in^3, i.e., specific gravity 7.09. The density of the liquid is assumed to be 0.0361 lb/in^3, i.e., specific gravity 1. See ref. [1]* for standard formulas and material values.

* Square-bracketed numbers refer to the bibliographic citations.

[1] Eshbach, C. Handbook of Engineering Fundamentals, c. 1952.

Other pertinent parameter values are shown in Table 1.1.

TABLE 1.1. INERTIAL CHARACTERISTICS OF A HYPOTHETICAL LIQUID-FILLED PROJECTILE IN CONFIGURATION A

Characteristic	Value	Dimension
volume of solid	703.7	in^3
weight of solid	180	lb
volume of liquid	554.2	in^3
weight of liquid	20	lb
total weight of projectile	200	lb
center of gravity of liquid, x_1	9.8	in
center of gravity of solid, x_2	16	in
center of gravity of projectile	15.38	in
axial moment of inertia of projectile	0.50506	slug ft^2
axial moment of liquid	0.019426	slug ft^2
axial moment of solid	0.48564	slug ft^2
pitch moment of inertia of projectile about its cg	3.8554	slug ft^2
pitch moment of liquid about x_1	0.14791	slug ft^2
pitch moment of solid about x_2	3.55813	slug ft^2

After exit from the cannon, the projectile will experience negligible axial acceleration, relative to its centripetal acceleration. Therefore, the free surface of the liquid will be a cylinder centered on the spin axis. This configuration, B, is shown in Figure 1.2.

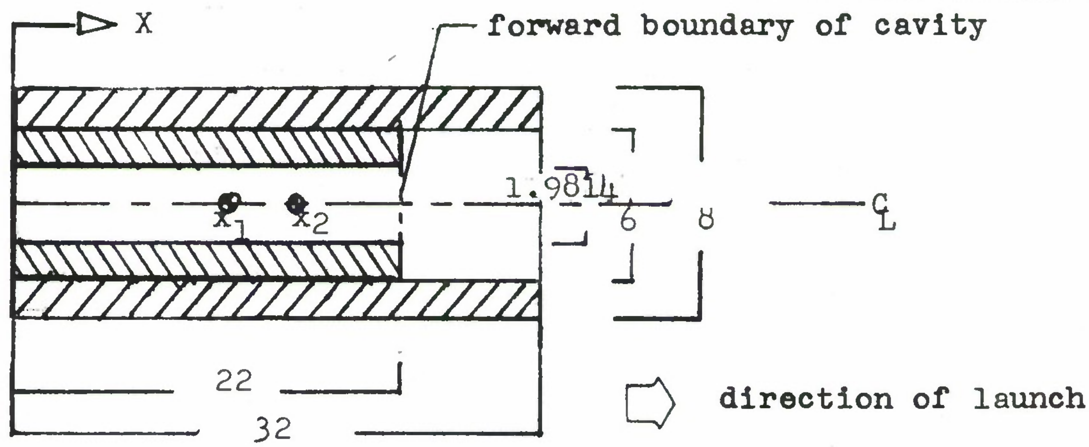

Figure 1.2. Configuration B

In Configuration B, the volumes and weights of solid and liquid are, of course, the same as in Configuration A. However, other inertial properties have changed from those shown in Table 1.1 to the values shown below:

center of gravity (cg) of liquid = 11.0 in

center of gravity (cg) of projectile = 15.5 in .

Thus, the projectile cg has moved forward by 0.12 in. In the M106 projectile, which has the same mass and caliber as this hypothetical projectile, the round-to-round standard deviation of cg position about the mean is 0.064 in., due to manufacturing tolerances [2]. Consequently, one may conclude that cg shifts due to liquid configuration changes in this

[2] Final Report of the HY-BRA Weapons System, Feb. 1970.

hypothetical projectile are less than two times the standard deviation associated with round-to-round cg shifts in comparable solid projectiles. Also

axial moment of inertia of liquid
(Configuration B) = 0.02154 slug ft^2

axial moment of inertia of projectile
(Configuration B) = 0.50718 slug ft^2 .

Further, comparisons of the tabular data show that the axial moment of inertia for Configuration B is 0.00212 slug ft^2 greater than for Configuration A. To put this difference in perspective, consider that the estimated round-to-round standard deviation in spin inertia for the M106 projectile, due to manufacturing variability, is 0.00252 slug ft^2 [2]. Thus, the change in spin inertia resulting from liquid configuration changes is less than one standard deviation of that of the comparable solid projectile. Using the deflection sensitivity to spin inertia given in Reference [2] for the M106 projectile, one expects a change in deflection of 0.26 mils due to change in inertial properties associated with the change in configuration.

Further,

pitch moment of inertia of liquid
(Configuration B) about x_1 = 0.18488 slug ft^2

pitch moment of inertia of projectile
(Configuration B) about its cg = 3.8401 slug ft^2 .

---

[2] Ibid.

Note that even though the pitch inertia of the liquid about its cg is greater in Configuration B than in Configuration A, the pitch moment of inertia of the whole projectile is smaller for Configuration B. This is due to the shift in cg of the liquid which brings it closer to the cg of the whole projectile. The difference in pitch inertia between the configurations is 0.0152 slug ft^2.

The estimated [2] round-to-round standard deviation of pitch inertia in the M106 projectile due to typical manufacturing tolerances is 0.0252 slug ft^2. Thus, liquid configuration change in the hypothetical projectile produces a change in pitch inertia only 0.6 of one standard deviation in pitch inertia of the M106.

One must conclude that the changes in inertial characteristics accompanying liquid configurational change in the hypothetical liquid-filled projectile would not be sufficient to produce significant changes in exterior ballistics, given only adequate ballistic stability of the liquid-filled projectile.

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[2] Ibid.

Estimate of Shape of the Free Surface of the Liquid in a Liquid-Filled Projectile During Acceleration

Consider Figure 1.3 below in which a cross section of the liquid-filled cavity of a projectile is depicted. Acceleration of the projectile is assumed in the positive X-direction.

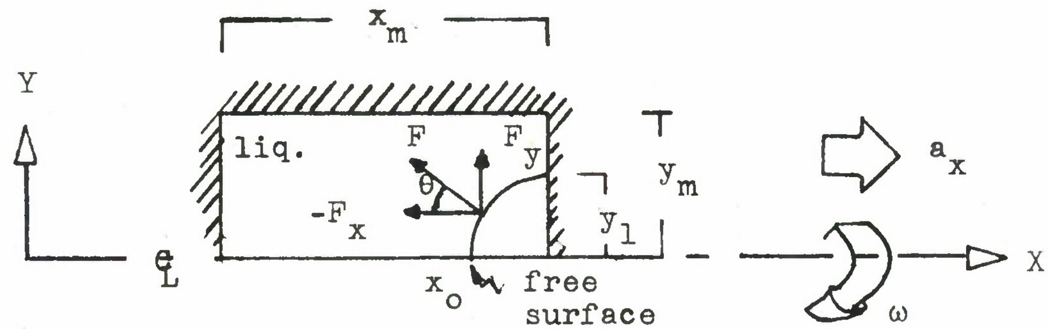

Figure 1.3. Liquid Free Surface During Acceleration

The free surface is defined by the function y(x). The differential element at (x,y), dv, experiences forces in the Y- and X-directions due to spin and setback, respectively. Neglecting gravity, which is small compared to the accelerations of interest, and assuming solid-body rotation, the forces on dv may be written as

$$F_x = - a_x \rho \, dv$$

$$F_y = \rho \, dv \, y \, \omega^2 \tag{1.1}$$

with

$$dv = 2 \pi y \, dy \, dx \quad .$$

The angular velocity, ω, is given in terms of the velocity of projectile, v_p, caliber, D, and twist, T, as

$$\omega = \frac{2 \pi v_p}{D T} \quad . \tag{1.2}$$

The vector sum of these forces on dv must be normal to the free liquid surface at equilibrium, since the liquid will not support shear stresses. Therefore, the slope of the free surface at (x,y) is given by

$$\frac{dy}{dx} = - (\tan \vartheta)^{-1} = \frac{-F_x}{F_y} = k \, y^{-1} \quad , \tag{1.3}$$

with

$$k = a_x \, \omega^{-2} \; .$$

Equation (1.3) may be integrated to yield

$$y^2 = 2 k (x - x_o)$$

or

$$y = [2 k (x - x_o)]^{\frac{1}{2}} \; , \tag{1.4}$$

and

$$x = x_o + \frac{y^2}{2 k} \; , \tag{1.5}$$

where the value of x_o is determined by the volume of the cavity and volume of liquid. **Since the volume, V, of liquid is assumed constant,**

$$V - \pi (y_m^2 - y_1^2) \, x_m = \int_o^{y_1} 2 \pi y \, x(y) \, dy, \tag{1.6}$$

$$0 \leq y_1 \leq y_m \; , \; 0 < x_o \leq x_m \; ,$$

with

$$y_1 = \min\{y_m,\ y(x_m) = [2\ k\ (x_m - x_o)]^{\frac{1}{2}}\} \quad . \qquad (1.7)$$

Then, from (1.5)

$$\frac{V}{2\ \pi} - (y_m^2 - y_1^2)\ \frac{x_m}{2} = \int_o^{y_1} \left(\frac{y^3}{2\ k} + x_o\ y\right) dy$$

$$\frac{V}{2\ \pi} - \frac{y_m^2\ x_m}{2} + (x_m - x_o)\ \frac{y_1^2}{2} - \frac{y_1^4}{8\ k} = 0 \quad .$$

For $y_1 < y_m$,

$$y_1^4 = \frac{4\ k}{\pi}\ (V_c - V) \qquad (1.8)$$

with

$$V_c = \pi\ x_m\ y_m^2$$

And for $y_1 = y_m$,

$$x_o = \frac{V}{\pi\ y_m^2} - \frac{y_m^2}{4\ k} \quad . \qquad (1.9)$$

<u>Example 1.</u>

Take the following conditions at peak projectile acceleration:

liquid volume, $V = 553.8\ in^3$ $(0.3205\ ft^3)$

cavity volume, $V_c = 622\ in^3$ $(0.36\ ft^3)$

projectile velocity, $v_p = 200$ ft/s

gun tube twist, $T = 20$ cal/rev

caliber, $D = 8$ in (2/3 ft)

projectile acceleration, $a_x = 4.504\ 10^5$ ft/s

$x_m = 22$ in (1.833 ft)

$y_m = 3$ in (0.25 ft) .

Then,

$$\omega = \frac{2\pi v_p}{D\,T} = 94.248 \text{ rad/sec (15 hz)} \quad ,$$

$$k = \frac{a_x}{\omega^2} = 50.70 \text{ ft} \quad ,$$

$$y^2 = 101.4\,(x - x_o) \quad .$$

Assuming

$$y_1 = y_m = 0.25 \text{ ft},$$

$$x(y_1) - x_o = 7\;10^{-3} \text{ in}$$

and

$$x_o = 19.587 \text{ in.}$$

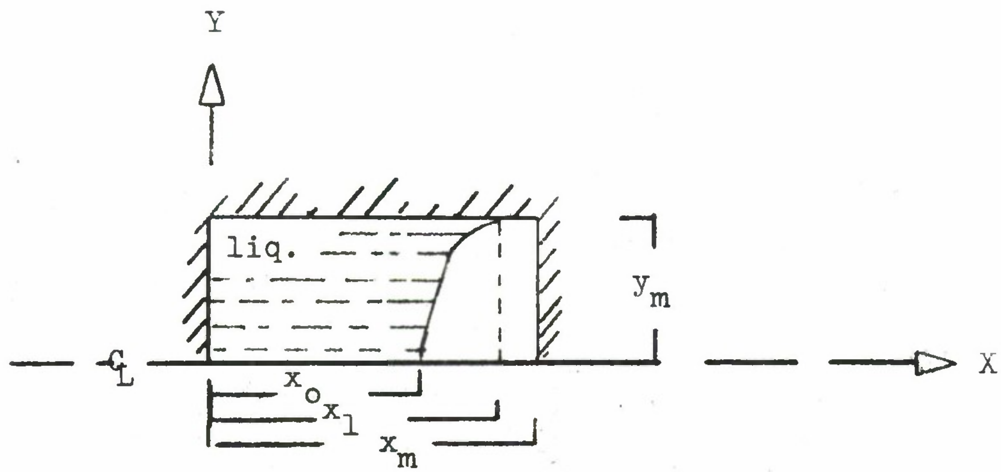

Figure 1.4. Liquid Free Surface During Acceleration

Figure 1.4 represents the position of the liquid surface where the unoccupied volume of the cavity is large. In this case the free (unoccupied) volume is

$$V_f = V_{cavity} - V_{liq} \quad . \tag{1.10}$$

From Figure 1.4,

$$V_f = \pi(x_m - x_1)\ y_m^2 + \int_{x_o}^{x_1} \pi\ y^2\ dx,\ 0 < x < x_m \tag{1.11}$$

Using (1.4, 1.11),

$$V_f = \pi(x_m - x_1)\ y_m^2 + \pi\ k(x_1 - x_o)^2 \quad . \tag{1.12}$$

But, from (1.5),

$$x_o = x_1 - \frac{y_m^2}{2\ k} \quad . \tag{1.13}$$

Then (1.12, 1.13) yield an expression for x_1.

$$x_1 = x_m + \frac{y_m^2}{4\ k} - \frac{V_f}{\pi\ y_m^2} \qquad (1.14)$$

Given the cavity dimensions and liquid volume, Equation (1.14) can be solved for x_1 and with this value x_o can be obtained from (1.13).

Example 2

Suppose that conditions at the muzzle of a gun are as follows:

projectile acceleration, a_x = 8.086 10^4 ft/s

projectile velocity, v_p = 2000 ft/s

twist, T = 20 cal/rev

caliber, D = 2/3 ft (8 in)

cavity free volume, V_f = 0.03935 ft^3

cavity length, x_m = 1.8333 ft

cavity radius, y_m = 0.25 ft

Using (1.2),

$$\omega = 942.48 \text{ rad sec}^{-1} \text{ (150 hz)} \quad .$$

From (1.3),

$$k = 0.09103 \text{ ft} \quad ,$$

and

$$y^2 = 0.18206\ (x - x_o)\ (\mathrm{ft})^2 \quad .$$

Then **(1.14) yields**

$$x_1 = 1.80457 \text{ ft} = 21.655 \text{ in} \quad ,$$

and from **(1.13),**

$$x_o = 1.46128 \text{ ft} = 17.535 \text{ in} \quad ,$$

so that

$$x_1 - x_o = 4.12 \text{ in.}$$

The radial acceleration at y_m in this example is a_y.

$$a_y(y_m) = y_m\ \omega^2 = .25(942.48)^2$$

$$a_y(y_m) = 22.2\ 10^4\ \mathrm{f/s^2}$$

$$\cong 0.69\ 10^4\ g \quad .$$

CHAPTER II

ANGULAR ACCELERATION OF THE LIQUID IN A LIQUID-FILLED PROJECTILE DURING FLIGHT

Liquid "Spinup" in Configuration A

We treat the position of the liquid within the shell as in Configuration A (Figure 1.1). Let the tangential or circumferential velocity of the liquid at radius r and time t be denoted by v. We neglect axial velocity components.

Consider the annular volume element shown in Figure 2.1.

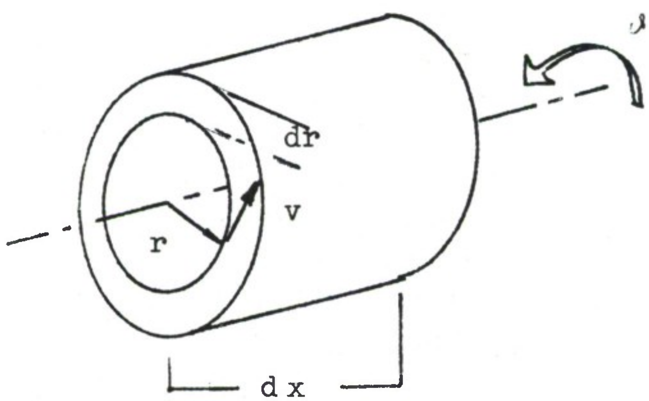

Figure 2.1. Volume Element Under Angular Acceleration

At position (r,x) in the liquid, the shear stress acting on the surface dA_- (below r) is

$$- \sigma_r(r,x)$$

where

$$dA_- = 2\pi r\, dx \quad .$$

At a radial position $r + dr$ the shear stress acting on the surface dA_+ is

$$\sigma_r(r+dr,x) \quad ,$$

where

$$dA_+ = 2\,\pi\,(r+dr)\,dx \quad .$$

Similarly at the left of the element the shear stress acting on the differential area dA_o is

$$-\,\sigma_x(r,x)$$

where

$$dA_o = 2\,\pi\,r\,dr \quad .$$

And at axial position $x + dx$, the shear stress acting on dA_o is

$$\sigma_x(r,x+dx) \quad .$$

The axial moment of inertia of the differential element is

$$I = 2\,\pi\,\rho\,r^3\,dx\,dr \quad . \tag{2.1}$$

The sum of all torques acting on the element is

$$M_\vartheta = [\sigma_x(r,x+dx) - \sigma_x(r,x)]\,dA_o\,r$$

$$+\,[\sigma_r(r+dr,x)\,dA_+ - \sigma_r(r,x)\,dA_-]\,r \quad . \tag{2.2}$$

Defining the angular velocity of the liquid at (r,x) at time t as $\omega(r,x,t)$, Newton's law gives

$$\dot{\omega} = M_\vartheta / I \quad , \tag{2.3}$$

where the functional dependence on r,x and time t has been suppressed notationally.

Then

$$\dot{\omega} = (\rho r)^{-1} [\sigma_x(r,x+dx) - \sigma_x(r,x)] / dx$$

$$+ (\rho r^2)^{-1} [\sigma_r(r+dr,x)(r+dr) - \sigma_r(r,x) r] / dr$$

Or in the limit as dx **and dr approach zero,**

$$\dot{\omega} = (\rho r)^{-1} \frac{\partial \sigma_x}{\partial x} + (\rho r^2)^{-1} \frac{\partial (r\sigma_r)}{\partial r} . \qquad (2.4)$$

But the dynamic viscosity η is defined such that

$$\sigma_x = \eta \, r \frac{\partial \omega}{\partial x}$$

$$\sigma_r = \eta \, r \frac{\partial \omega}{\partial r} \qquad (2.5)$$

Then with η taken independent of x,r

$$\dot{\omega} = (\rho r)^{-1} \eta \, r \frac{\partial^2 \omega}{\partial x^2} + (\rho r^2)^{-1} \eta \frac{\partial}{\partial r} (r^2 \frac{\partial \omega}{\partial r}) .$$

$$\dot{\omega} = \nu \frac{\partial^2 \omega}{\partial x^2} + \nu r^{-2} [2 r \frac{\partial \omega}{\partial r} + r^2 \frac{\partial^2 \omega}{\partial r^2}]$$

where ν is the kinematic viscosity.

$$\nu = \eta \rho^{-1} \qquad (2.6a)$$

$$\dot{\omega} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + 2 r^{-1} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial r^2}\right] \qquad (2.6b)$$

If the axial variability in ω can be neglected, the simplified result in one spacial dimension is

$$\dot{\omega} = \nu \left[2\, r^{-1} \frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial r^2} \right] \quad . \tag{2.7}$$

The tangential velocity v in the liquid at radius r is

$$v = r\, \omega \quad , \tag{2.8}$$

with v a function of r and time t,

$$v = w(r,t) \quad . \tag{2.9}$$

With (2.7, 2.8)

$$\dot{w} = \nu \frac{\partial^2 w}{\partial r^2}$$

or

$$w_t - \nu\, w_{rr} = 0 \quad , \tag{2.10}$$

where the subscripts indicate partial differentiation with respect to that variable. Note that for stationary conditions, i.e., zero w_t, w_r is a constant c and $w(r) = c\, r$.

Notation can be simplified by defining a dimensionless radius ξ and dimensionless time τ.

Let

$$\xi = r/r_1$$

and

$$\tau = t\, \nu \,/\, r_1^2 \tag{2.11}$$

with r_1, ν constants.

Then Equation (2.10) becomes

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} \quad , \quad 0 < \xi < 1 \qquad (2.12)$$

$$, \quad \tau > 0$$

where

$$u(\xi,\tau) = w(r,t) \quad . \qquad (2.13)$$

The boundary conditions for (2.12) are

$$u(\xi,0) = 0 \quad , \quad 0 \leq \xi \leq 1 \qquad (2.14)$$

$$u(0,\tau) = 0 \quad , \quad \tau > 0 \qquad (2.15)$$

$$u(1,\tau) = \varphi(\tau)$$

with

$$\varphi(\tau) = f(t) \quad , \quad t > 0 \qquad (2.16)$$

$$f(t) = f_1(t) + f_2(t)$$

$$f_1(t) = a\,t \quad , \quad t \geq 0$$

$$f_2(t) = 0 \quad , \quad t \leq t_1$$

$$f_2(t) = -\,a(t - t_1) \quad , \quad t > t_1 \quad . \qquad (2.17)$$

Thus

$$\varphi(\tau) = \varphi_1 + \varphi_2$$

$$\varphi_1(\tau) = \alpha\,\tau \quad , \quad \tau \geq 0$$

$$\varphi_2(\tau) = 0 \quad , \quad \tau \leq \tau_1$$

$$\varphi_2(\tau) = -\alpha(\tau - \tau_1) \quad , \quad \tau > \tau_1 \quad , \tag{2.18}$$

with

$$\alpha = \frac{r_1^2}{\nu}\,a$$

$$\tau_1 = t_1\,\nu\,/\,r_1^2 \quad . \tag{2.19}$$

Taking the Laplace transform of (2.12)

$$s\,u^*(\xi,\,s) - u(\xi,\,0) = u^*_{\xi\xi}(\xi,\,s)$$

or from (2.14)

$$s\,u^*(\xi,\,s) = u^*_{\xi\xi}(\xi,\,s) \quad . \tag{2.20}$$

Also, from (2.15, 2.16),

$$u^*(0,\,s) = 0 \quad ,$$

and

$$u^*(1,\,s) = \varphi^*(s) \quad , \tag{2.21}$$

or

$$\varphi^*(s) = \alpha\,s^{-2} - \alpha\,s^{-2}\,e^{-\tau_1 s} \quad . \tag{2.22}$$

Since $u^*(\xi,\,s)$ is not a function of time, the partial derivatives in Equation (2.20) are actually total derivatives of u^* with respect to ξ.

Thus,

$$\frac{d^2 u^*}{d \xi^2} - s u^* = 0 \tag{2.23}$$

whose solution with initial conditions given by (2.21) is

$$u^*(\xi,s) = \varphi^*(s) \frac{\sinh \sqrt{s}\, \xi}{\sinh \sqrt{s}} \quad . \tag{2.24}$$

Using a series expansion of

$\sinh \sqrt{s}\, \xi \ / \ \sinh \sqrt{s}$

-- see p. 139, Churchill [3] -- and applying the convolution theorem

$$u(\xi,\tau) = \frac{2}{\sqrt{\pi}} \sum_{n=o}^{\infty} \{ \int_{\ell_1(n,\xi)}^{\infty}$$

$$\varphi(\tau - (2 n + 1 - \xi)^2 / (4 \lambda^2))\, e^{-\lambda^2}\, d\lambda$$

$$- \int_{\ell_2(n,\xi)}^{\infty} \varphi(\tau - (2 n + 1 + \xi)^2 / (4 \lambda^2))$$

$$e^{-\lambda^2}\, d\lambda \} \quad ,$$

with

$$\begin{aligned} \ell_1(n,\xi) &= \frac{2 n + 1 - \xi}{2 \sqrt{\tau}} \\ \ell_2(n,\xi) &= \frac{2 n + 1 + \xi}{2 \sqrt{\tau}} \quad . \end{aligned} \tag{2.25}$$

---

[3] **Churchill, R.V. <u>Operational Mathematics</u>, 2nd Ed., McGraw-Hill, New York, c. 1958.**

For the special case of $\varphi(\tau)$ a constant φ_o

$$u(\xi,\ \tau) = \varphi_o \sum_o^\infty \left[\operatorname{erf}\left(\frac{2\ n + 1 + \xi}{2\sqrt{\tau}}\right) \right.$$

$$\left. - \operatorname{erf}\left(\frac{2\ n + 1 - \xi}{2\sqrt{\tau}}\right)\right] \quad . \tag{2.26}$$

Substitution of (2.22) into (2.25) appears to be too complex to pursue. A numerical approach starting with Equation (2.10) and boundary conditions given by (2.14, 2.15, 2.16) was judged more profitable.

For low viscosity liquids and rapid rise of driving angular velocity to a nearly constant level, the result in (2.26) may be a reasonable description. This expression was evaluated for unity φ_o and plotted in **Figure** 2.2. Generally it was possible to truncate the series at four terms or less to achieve 0.1% accuracy.

The tangential velocity of the liquid in Configuration A is also shown as a function of τ for several radial positions in **Figure** 2.3.

Because angular acceleration of the liquid varies with position, it is useful to define an effective angular velocity as that value which would give the liquid angular momentum if distributed uniformly thruout the liquid.

Thus, for the nondimensional case,

$$\bar{\omega}_A = \frac{\int_o^1 \xi\ [u(\xi,\tau)\ \xi^{-1}]\ d\xi}{\int_o^1 \xi\ d\xi}$$

$$\bar{\omega}_A(\tau) = 2\int_o^1 u(\xi,\tau)\ d\xi \quad . \tag{2.27}$$

A plot of $\bar{\omega}_A$ versus τ is shown in **Figure** 2.4.

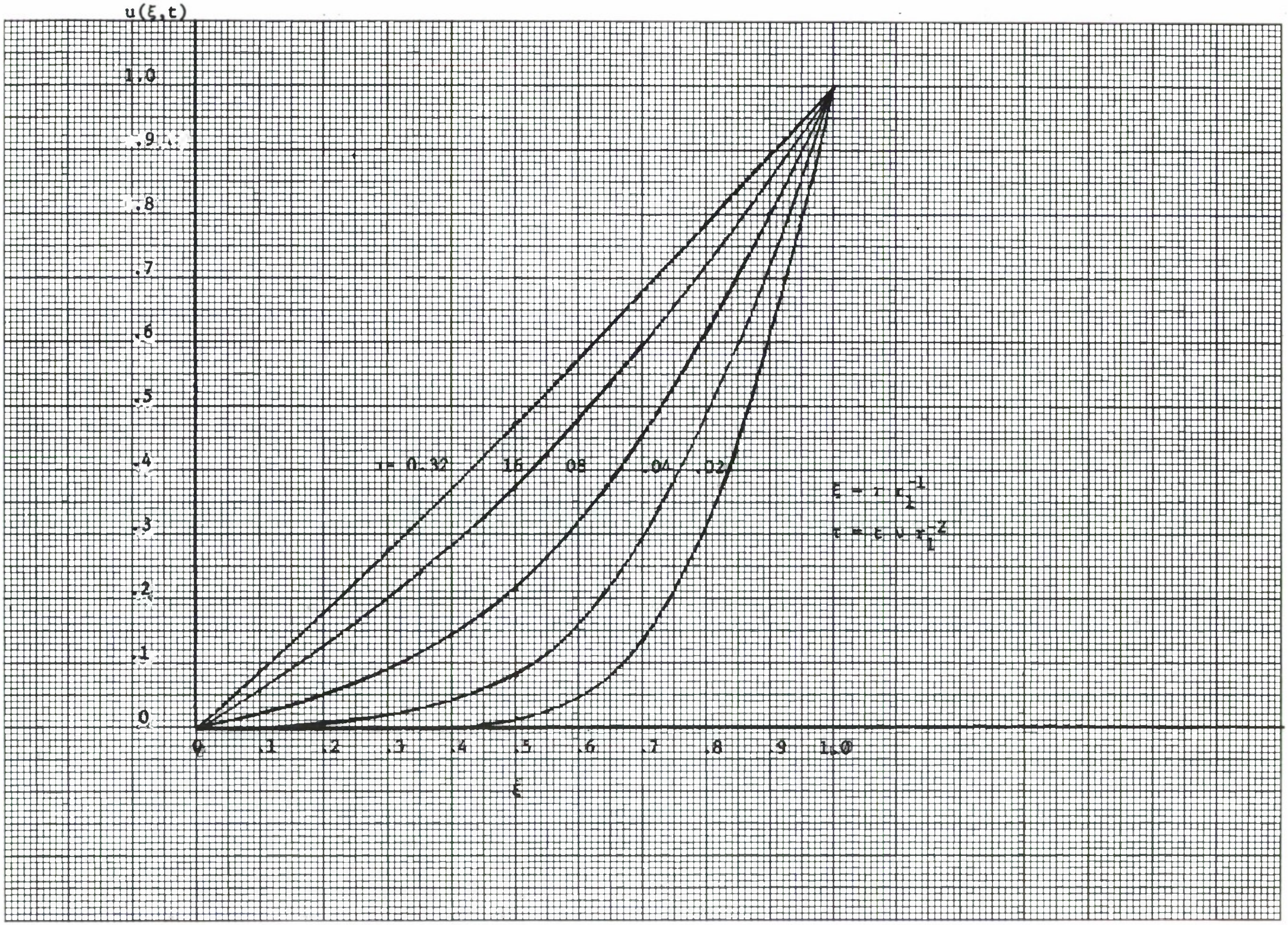

Figure 2.2. Tangential Velocity of Liquid Versus Radial Position at Several Values of Time (Liquid Configuration A)

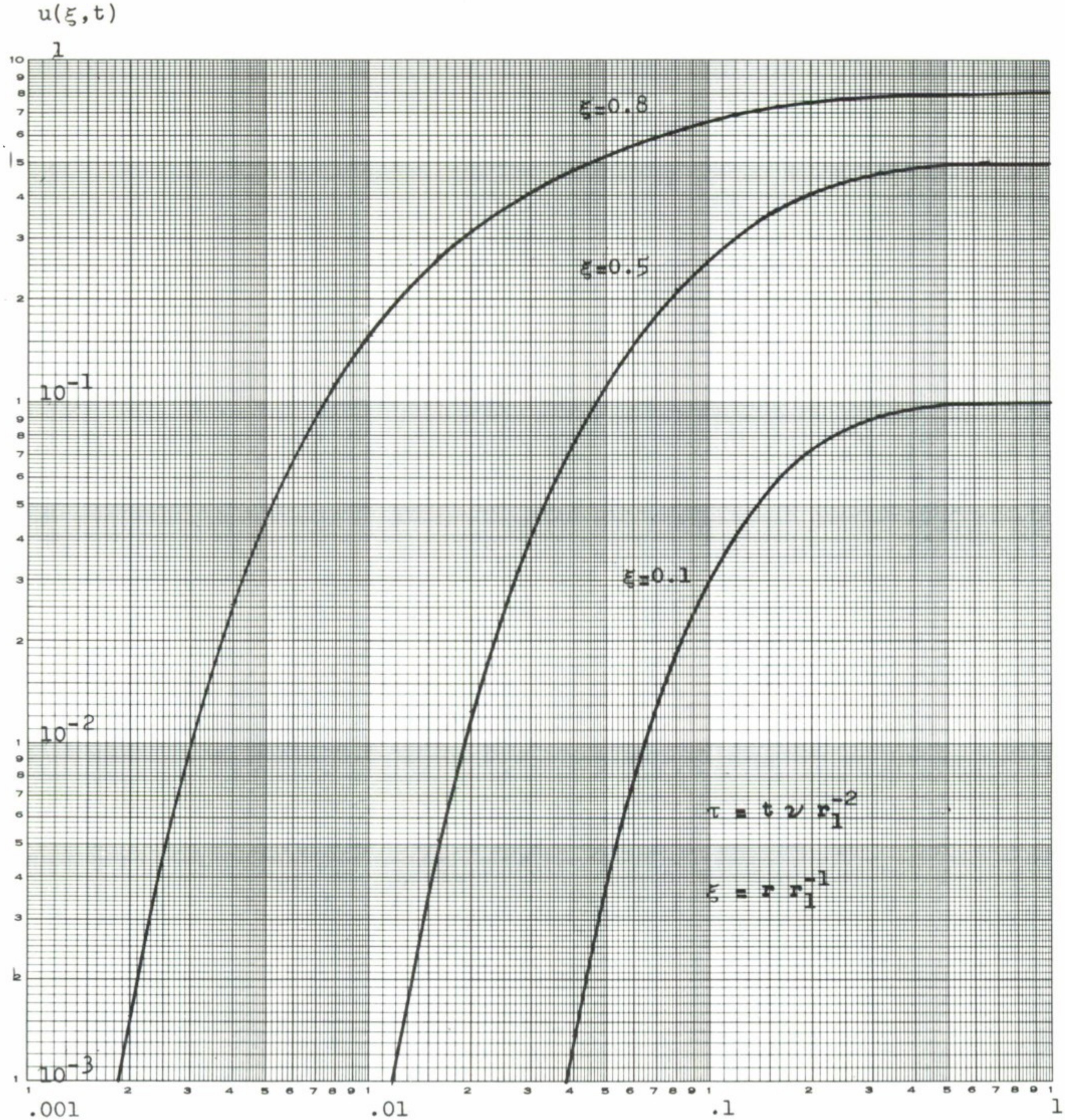

Figure 2.3. Tangential Velocity of Liquid Versus Time for Three Radial Positions

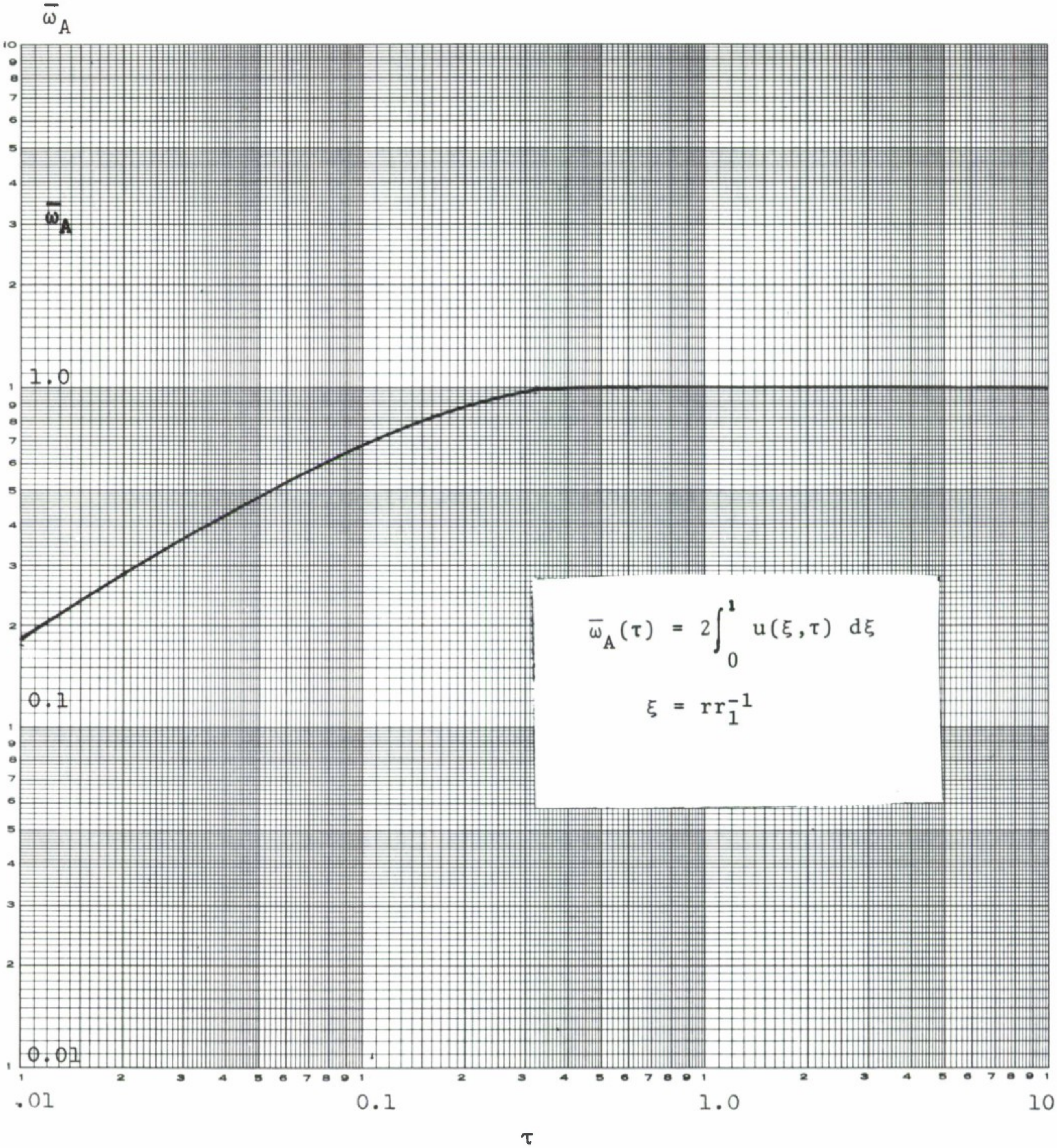

Figure 2.4. Effective Angular Velocity of Liquid in Configuration A Versus Time

Figure 2.3 shows that a nondimensional time τ of 0.1 is required to achieve 50% or more of the asymptotic velocity for $\xi > 0.5$. To apply these results to a specific example, suppose that

$$\nu = 10 \text{ centistokes } (0.1 \text{ stoke})$$

$$r_1 = 7.62 \text{ cm } (3 \text{ in}) \quad .$$

Then if "spinup" is arbitrarily defined as the time at which the angular kinetic energy reaches 70% of its asymptotic value*, a time corresponding to $\tau = 0.125$ is required. This time

$$t = \tau \, r_1^2 \, \nu^{-1}$$

$$= 0.125 \, (7.62)^2 \, 10$$

$$t = 72.6 \text{ sec} \quad .$$

Since this time exceeds the maximum time of flight, one would not expect spinup to occur in a liquid-filled projectile of this type having the above viscosity and remaining in Configuration A.**

Further during the inbore period, taken as 14 milliseconds, essentially no rotation of the liquid occurs since this time corresponds to τ of $2.4 \; 10^{-5}$ for the above example.

In order for the liquid to remain in a stable Configuration B after launch, the minimum radial acceleration must

---

* Approximately 75% of asymptotic angular momentum.

**As indicated in the Introduction, the conditions for this behavior do not obtain in the XM736 projectile, where experimental data indicate that spinup is complete within one to five seconds.

be at least one g. Thus, the limiting angular velocity of liquid for Configuration B is 19.74 rad/sec; and for a case spin of 942.5 rad/sec, implies a limiting value of $\bar{\omega}_A$ of approximately 0.02. **From Figure 2.4 one notes that this** value of $\bar{\omega}_A$ is obtained at

$$\tau = 0.012 \quad ,$$

which, for this example, corresponds to a time

$$t = \tau \, r_1^2 \, \nu^{-1}$$

$$t = 0.012 \, (7.62)^2 \, 10$$

$$t \cong 7 \text{ sec} \quad .$$

Beyond the time at which Configuration B is attained, the spin dynamics of the liquid must be obtained from the solution of the following boundary-value problem.

Angular Acceleration of the Liquid in Configuration B

As previously developed, the governing differential equation is

$$\omega_t = \nu \, [\omega_{xx} + 2 \, r^{-1} \, \omega_r + \omega_{rr}] \quad .$$

Letting

$$u(\xi,\lambda,\tau) = \omega(r,x,t) \quad ,$$

with

$$\xi = r/r_1 \ , \quad \xi_o = r_o/r_1 \ ,$$

$$\lambda = x/r_1 \ , \quad \lambda_1 = x_m/r_1 \ , \qquad \text{and}$$

$$\tau = t\,\nu/r_1^2 \ , \tag{2.28}$$

$$u_\tau = u_{\lambda\lambda} + 2\,\xi^{-1}\,u_\xi + u_{\xi\xi} \ , \quad \xi_o < \xi < 1 \tag{2.29}$$

$$0 < \lambda < \lambda_1$$

$$\tau > 0 \ .$$

The initial and boundary conditions in this case are

$$u(\xi,\lambda,0) = \psi(\xi,\lambda) \quad , \quad \xi_o < \xi < 1$$

$$u_\xi(\xi_o,\lambda,\tau) = 0 \ ^{*} \qquad 0 < \lambda < \lambda_1$$

$$u(1,\lambda,\tau) = \varphi(\tau) \qquad 0 \leq \lambda \leq \lambda_1$$

$$u(\xi,0,\tau) = \varphi(\tau)$$

$$u(\xi,\lambda_1,\tau) = \varphi(\tau) \ , \qquad \tau > 0 \ . \tag{2.30}$$

Note τ equal to zero corresponds to the time at which Configuration B is realized.

We particularize the functions $\psi(\xi)$ and $\varphi(\tau)$ as follows:

---

* At a free surface the shear stress in the liquid must vanish.

$$\psi(\xi,\lambda) = \bar{\omega}_{lim} \tag{2.31}$$

$$\varphi(\tau) = 1 \quad \text{(Case 1)} \quad , \text{ or,}$$

$$\varphi(\tau) = (1.2 + c\tau)^{-0.2808} \quad \text{(Case 2)} \quad ,$$

with

$$\bar{\omega}_{lim} \text{ and } c \text{ constants .} \tag{2.32}$$

The limiting or minimum angular velocity for Configuration B is $\bar{\omega}_{lim}$. The form of $\varphi(\tau)$ for Case 2 is motivated by the despin characteristics of a typical eight-inch projectile. A generalization of this formula is developed in the Appendix.

In the case considered

$$\xi_o = 0.33$$

$$\bar{\omega}_{lim} = 0.02$$

$$c = 0.0288 \, r_1^2 \, \nu^{-1} \tag{2.33}$$

$$r_1 = 7.62 \text{ cm}$$

$$\nu = 0.1 \text{ and } 0.01 \text{ stoke} \quad .$$

It is convenient to obtain a numerical solution to this problem thru spacial discretization.

The interval in ξ $(\xi_o, 1)$ is divided into n-1 equal segments of length

$$\Delta\xi = (1-\xi_o) \; / \; (n-1) \quad . \tag{2.34}$$

Similarly, the interval in λ $(0,\lambda_1)$ is divided into m-1 equal segments of length

$$\Delta\lambda = \lambda_1 / (m-1) \quad . \tag{2.35}$$

We define the value of u at the nodal points of this segmented space as follows

$$u_{ji} = u_{ji}(t) = u(\xi_j,\lambda_i,t) \quad , \quad 1 \le j \le n \quad , \quad 1 \le i \le m \quad , \tag{2.36}$$

with

$$\xi_j = \xi_o + (j-1)\,\Delta\xi$$

$$\lambda_i = (i-1)\,\Delta\lambda \quad .$$

Division of the space is illustrated in Figure 2.5.

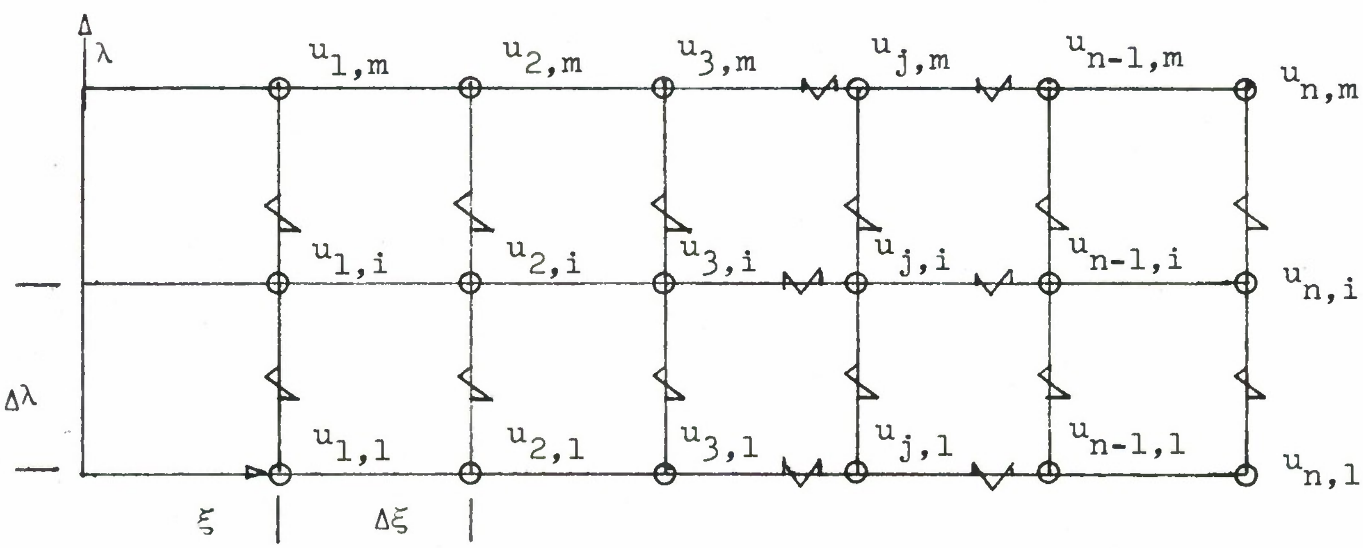

Figure 2.5. Nodal Points for Finite-Difference Method

The second central-difference approximation is used to approximate $u_{\xi\xi}$ and $u_{\lambda\lambda}$ as follows:

$$u_{\xi\xi}(\xi_j,\lambda_i) \cong (\Delta\xi)^{-2}\,(u_{j+1,i} - 2\,u_{j,i} + u_{j-1,i}) \quad ,$$

$$u_{\lambda\lambda}(\xi_j,\lambda_i) \cong (\Delta\lambda)^{-2}\,(u_{j,i+1} - 2\,u_{j,i} + u_{j,i-1}) \quad ,$$

$$1 < j < n$$
$$1 < i < m \quad . \tag{2.37}$$

A central-difference approximation is also used for u_ξ.

$$u_\xi(\xi_j,\lambda_i) \cong \frac{u_{j+1,i} - u_{j-1,i}}{2\,\Delta\,\xi} \quad , \tag{2.38}$$

$$1 < j < n$$
$$1 < i < m \quad .$$

With these approximations, Equation (2.29) is approximated as

$$\frac{d}{d\tau}\,u_{j,i} = (\Delta\xi)^{-2}\,[(1-\Delta\xi/\xi_j)\,u_{j-1,i} - 2\,u_{j,i}$$
$$+ (1+\Delta\xi/\xi_j)\,u_{j+1,i}] +$$
$$(\Delta\lambda)^{-2}\,[u_{j,i-1} - 2\,u_{j,i} + u_{j,i+1}] \quad ,$$

$$1 < j < n$$
$$1 < i < m \quad . \tag{2.39}$$

At the boundaries of the discrete space, from (2.31, 2.32)

$$\frac{d}{d\tau} u_{j,1} = \varphi_\tau = - 0.2808\ c\ (1.2 + c\ \tau)^{-1.2808} ,$$

$$\frac{d}{d\tau} u_{j,m} = \varphi_\tau , \quad 1 \leqslant j \leqslant n ,$$

and,

$$\frac{d}{d\tau} u_{n,i} = \varphi_\tau , \quad 1 \leqslant i \leqslant m . \tag{2.40}$$

For Case 1, c is set to zero.

$$\frac{d}{d\tau} u_{1,i} = (\Delta\xi)^{-2} \frac{[(\xi_2/\xi_1)^2 + 1]}{2} (u_{2,i} - u_{1,i})$$

$$+ (\Delta\lambda)^{-2} [u_{1,i-1} - 2\ u_{1,i} + u_{1,i+1}] ,$$

$$1 < i < m . \tag{2.41}$$

The first term on the r.h.s. of (2.41) derives from the fact that the net torque due to radial gradients acting on the mass element within the annular segment ($r_1 \leqslant r < r_2$) produces an angular acceleration

$$\dot{\omega}_1 = (\rho r_1^2)^{-1} \frac{(r_2\ \sigma_2 + r_1\ \sigma_1)}{2\ \Delta\ r} ,$$

and, for

$$\sigma_2 \cong \eta\ r_2 \frac{(\omega_2 - \omega_1)}{\Delta\ r} ,$$

$$\sigma_1 = \eta\, r_1 \frac{(\omega_2 - \omega_1)}{\Delta r} \quad ,$$

$$\dot{\omega}_1 = \nu\, (\Delta r)^{-2} \frac{[1 + (r_2/r_1)^2]}{2} (\omega_2 - \omega_1) \quad .$$

This expression is equivalent to the first term of the r.h.s. of (2.41).

The initial conditions for Cases 1 and 2 (2.31, 2.33) are:

$$u_{j,1} = u_{j,m} = 1,\ .944,\ 1 \leq j \leq n$$

$$u_{n,i} = 1,\ .944,\ 1 \leq i \leq m$$

$$u_{j,i} = \bar{\omega}_{1im} = 0.02 \quad , \quad 1 < j < n$$
$$1 < i < m \quad . \tag{2.42}$$

Note that the initial projectile spin is normalized to unity. The actual liquid spin may be obtained from the relation

$$\omega(\tau) = \omega_o\, u(\xi,\lambda,\tau) \quad , \tag{2.43}$$

where ω_o is the projectile spin at launch. **At the time Configuration B is achieved, the normalized spin is taken as 0.944. This time is approximately 7 seconds after launch in the examples treated here.**

Because of the relatively weak longitudinal gradients in the angular velocity relative to the radial gradients, the longitudinal (λ) dependence was neglected in the numerical results **presented here.***

With no dependence on the index i, Equation (2.39) can be simplified as

* **Check runs considering λ-dependence show essentially equivalent results.**

$$\frac{d}{d\tau} u_j = (\Delta\xi)^{-2} [(1-\Delta\xi/\xi_j) u_{j-1} - 2 u_j$$

$$+ (1+\Delta\xi/\xi_j) u_{j+1}] \quad , \quad 1 < j < n \quad . \qquad (2.44)$$

And, from (2.40, 2.41)

$$\frac{d u_n}{d\tau} = 0 \qquad \text{(Case 1)}$$

$$\frac{d u_1}{d\tau} = (\Delta\xi)^{-2} \frac{[(\xi_2/\xi_1)^2 + 1]}{2} (u_2 - u_1) \quad . \qquad (2.45)$$

Using values of

$$n = 11 \quad ,$$

$$\xi_o = 0.33 \quad ,$$

$$\Delta\xi = 0.067 \quad ,$$

and a finite time step $\Delta\tau = 10^{-4}$, (2.44, 2.45) were integrated using a fourth-order Runge-Kutta procedure. The numerical results are illustrated in **Figure** 2.6.

The equivalent angular velocity of the liquid can also be obtained for Configuration B. This is obtained numerically as follows:

$$\bar{\omega}_B = \frac{\sum_{j=1}^{n-1} (\xi_j + \xi_{j+1})(u_j + u_{j+1})}{2 \sum_{j=1}^{n-1} \xi_j + \xi_{j+1}} \quad . \qquad (2.46)$$

This result is plotted in **Figure** 2.7.

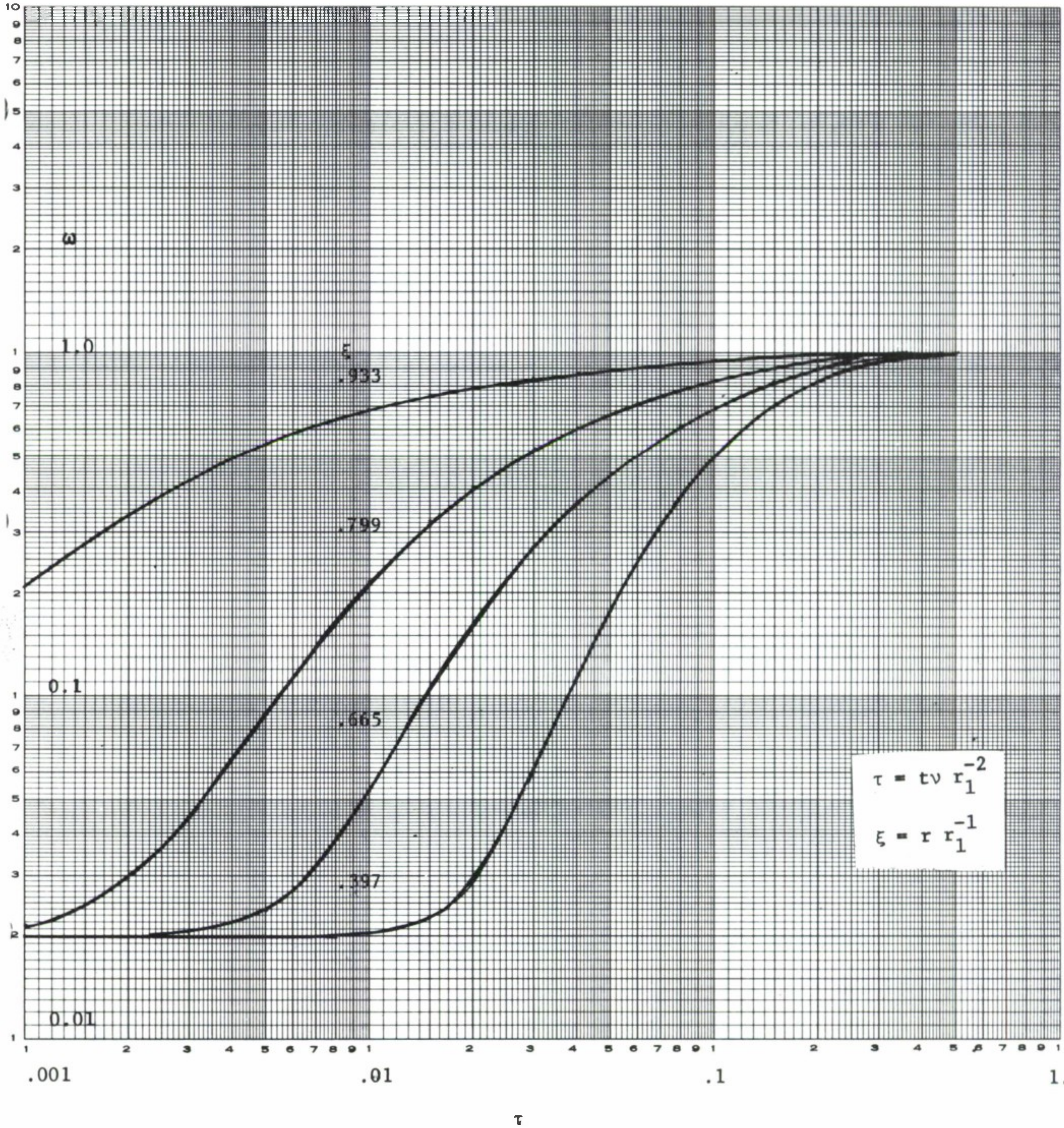

Figure 2.6. Angular Velocity of Liquid Versus Time for Several Radial Positions (Liquid Configuration B)

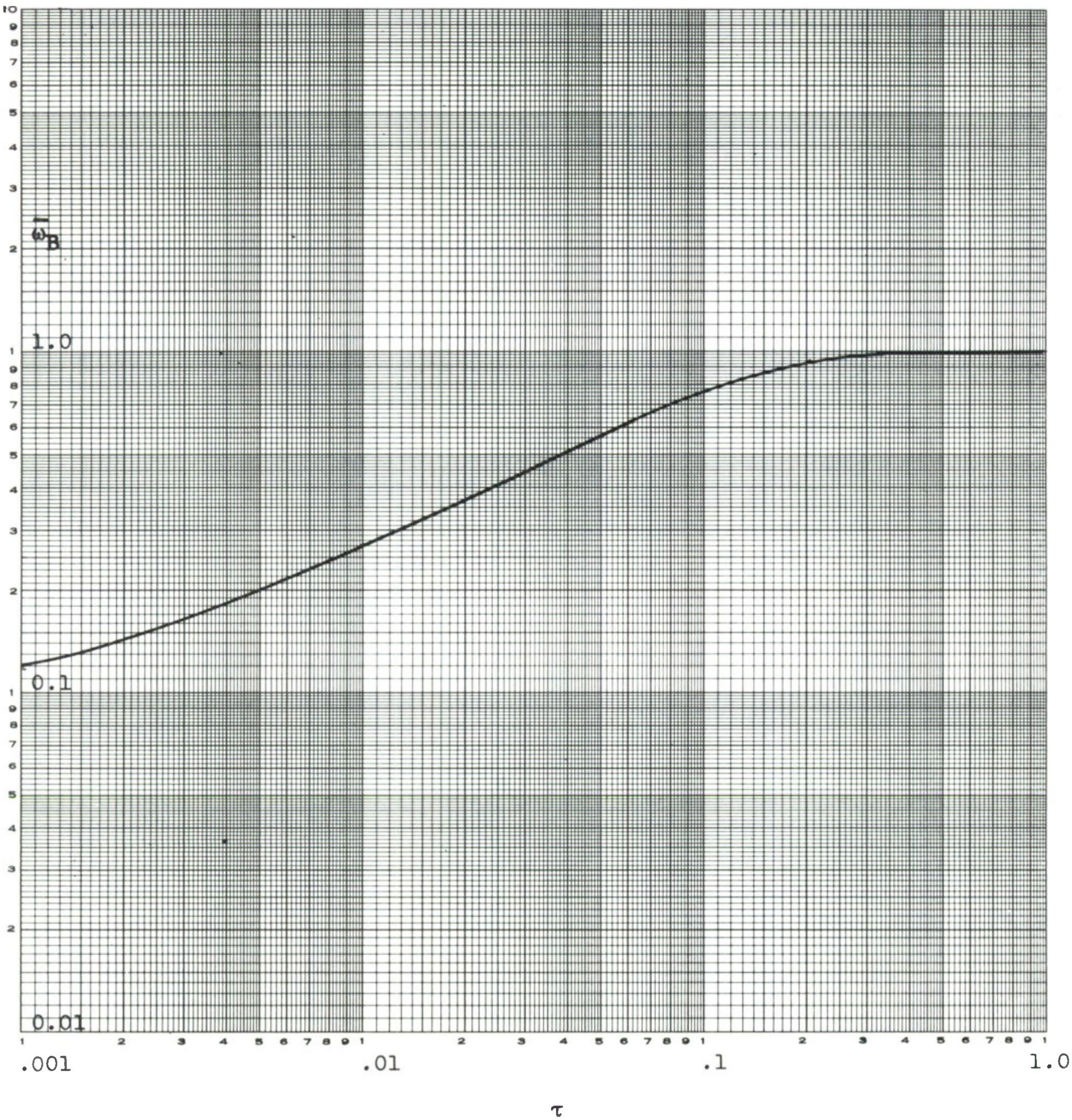

Figure 2.7. Effective Angular Velocity of Liquid in Configuration B Versus Time

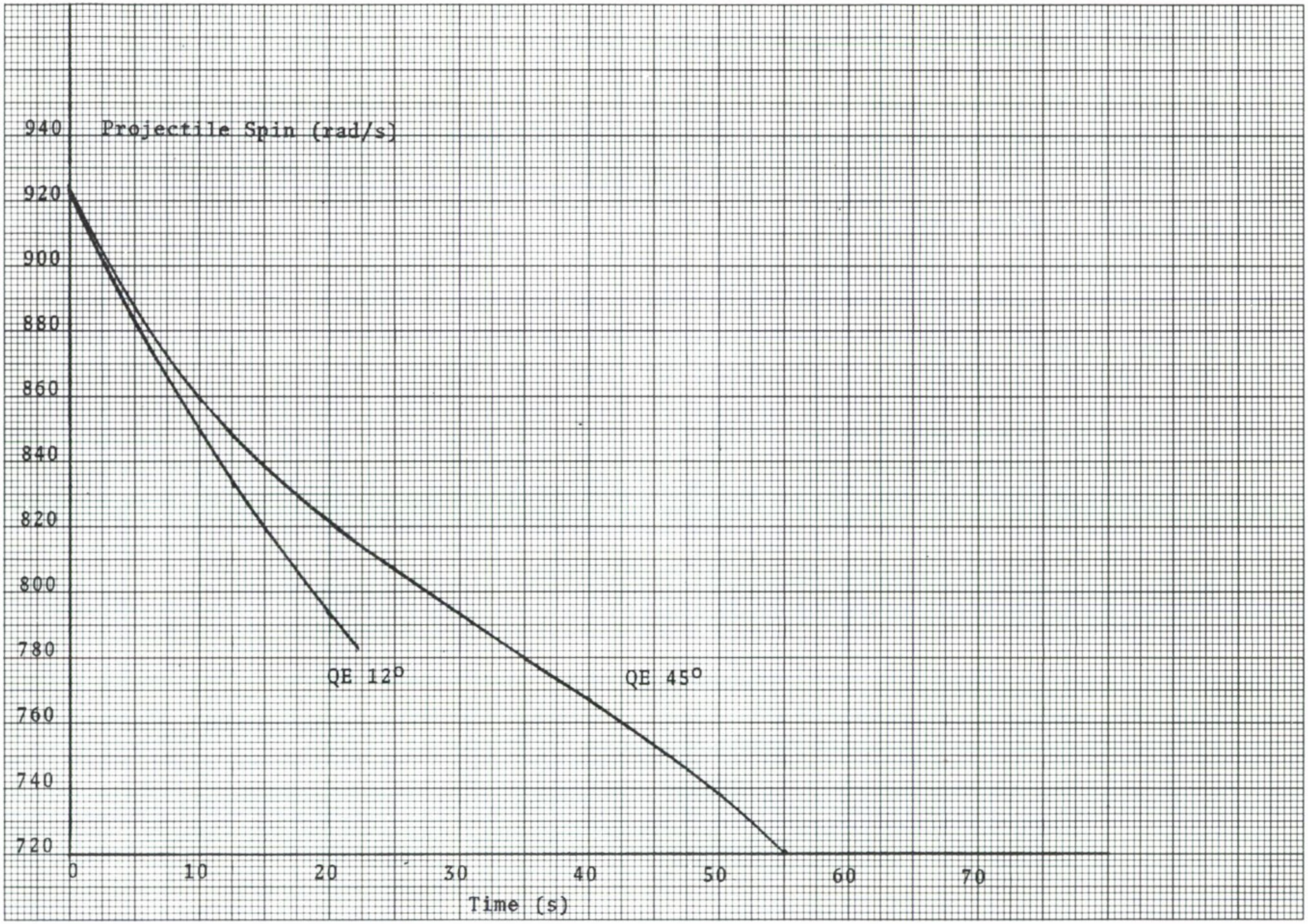

Figure 2.8. Spin Versus Time for a Modified M509 Projectile at Zone 7 from the XM201 Cannon (Inertial Characteristics of the XM736 Projectile)

Based upon the run down of spin over time for an M509 projectile type, a second numerical example was examined (Case 2). The spin versus time for a modified M509 projectile is shown in Figure 2.8. The boundary conditions at ξ_n for Case 2 -- Equation (2.40) -- closely match the spin characteristics of the M509. Consequently, this example is a reasonable simulation of the spin dynamics of the liquid in the XM736 projectile. As indicated in Equation (2.33), two values of liquid viscosity were used for Case 2 -- 1 and 10 centistokes. Because of the strong dependence of viscosity upon temperature, this viscosity range is necessary to encompass the expected range of liquid temperature. In this example a launch spin ω_o of 923 rad/sec is assumed. Numerical results are shown in Figure 2.9.

At this point it is necessary to repeat the caveat that the above analysis of the dynamics of liquid spinup applies, strictly, only to liquid-filled projectiles having long, narrow cavities and quite viscous fills so that laminar flow obtains. In the XM736, these conditions do not hold (even at 10 centistokes). However, due to the brevity of the launch period, negligible liquid spinup is expected in the XM736 so that the angular momentum of the projectile is about 4% less than that of a comparable solid projectile having the same total mass and exterior configuration. Thus, in terms of solid-projectile behavior, the effective axial moment of inertia of the XM736 is approximately 4% less than its solid counterpart. Pitch inertia is negligibly affected by liquid rotation in this system, as previously shown.

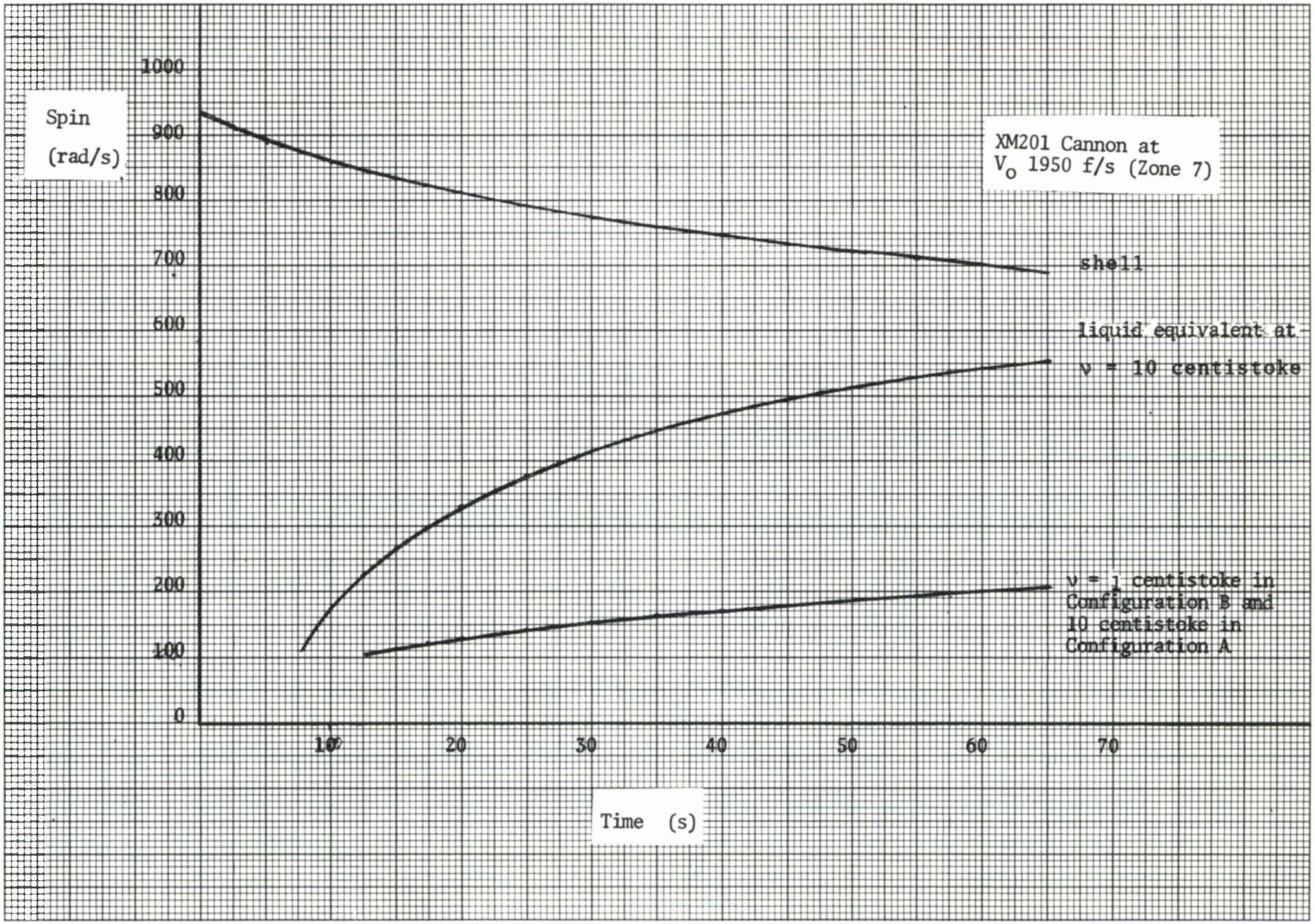

Figure 2.9. Estimated Spin Versus Time for the Liquid in a Type XM736 Projectile at Two Values of Viscosity

Exterior Ballistic Differential Effects

An attempt to assess the exterior ballistic effects of the indicated change in rotational inertia relative to a "ballistically matched" solid projectile was made in the following manner. A set of runs was made with a modified point-mass program [4] in which the inertial characteristics of a projectile having characteristics similar to the M106 projectile, and termed the standard eight-inch projectile, were changed incrementally as shown below in Table 2.1. Shifts in range and deflection relative to those of the standard projectile are noted. Projectile characteristics are shown in Table 2.4.

The combined effect of movement of the center of gravity toward the nose by 0.12 inch and reduction in the axial moment of inertia by 4% is to change the deflection by about 1.36 milliradians. This magnitude is less than one deflection probable error for the M106 system [2,5]. Change in range is negligible. Similar ballistic sensitivities for the M106 and M509 projectiles are shown in Tables 2.2, 2.3A, and 2.3B.

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[2] Op. Cit.

[4] Lieske, R.F., and Reiter, M.L. Equations of Motion for a Modified Point Mass Trajectory, BRL Report No. 1314, Ballistics Research Labs, Aberdeen, Md., March 1966.

[5] Firing Tables for Cannon, 8-Inch Howitzer, M2, M2A1 and M47 Firing Projectile, HE, M106, FT 8-J-3, Hdqts Dept of Army, October 1960.

TABLE 2.1. BALLISTIC SENSITIVITY OF A STANDARD PROJECTILE IN M2 8-INCH HOWITZER AT MAXIMUM RANGE TO CHANGES IN CG POSITION AND AXIAL MOMENT OF INERTIA

Loc. CG (cal)	Axial MI (slug ft^2)	Range (m)	Δ Range (m)	Defl. (m)	Δ Defl. (m)
2.5	0.5	16,943	0	665	0
2.485	0.5	16,943	0	672	7
2.5	0.48	16,945	2	634	- 31
2.485	0.48	16,945	2	642	- 23

TABLE 2.2. BALLISTIC SENSITIVITY OF THE M106 PROJECTILE IN M2 HOWITZER TO A JOINT CHANGE IN CG POSITION AND AXIAL MOMENT OF INERTIA

QE 45°

V_o (f/s)	Loc. CG (cal)	Axial MI (kg m^2)	Range (m)	Defl. (m)	Δ Defl. (m)
1380	2.840	0.553	11751	210	0
(Z5)	2.825	0.530	11752	201	- 9
1950	2.840	0.553	16795	417	0
(Z7)	2.825	0.530	16796	400	- 17

TABLE 2.3A. BALLISTIC SENSITIVITY OF THE M509 PROJECTILE IN M2 AND M110E2 HOWITZERS TO A JOINT CHANGE IN CG POSITION AND AXIAL MOMENT OF INERTIA

QE 45°

V_o (f/s)	Loc. CG (cal)	Axial MI (kg m^2)	Range (m)	Defl. (m)	Δ Defl. (m)
	M2 Howitzer				
1906	3.592	0.570	17059	298	0
(Z5)	3.577	0.547	17059	285	- 13
	M110E2 Howitzer				
1040	3.592	0.570	8547	136	0
(Z3)	3.577	0.547	8548	130	- 6
1960	3.592	0.570	17622	398	0
(Z7)	3.577	0.547	17622	382	- 16
2440	3.592	0.570	22853	621	0
(Z9)	3.577	0.547	22853	595	- 26

TABLE 2.3B. BALLISTIC SENSITIVITY OF THE M509 PROJECTILE IN THE M110E2 HOWITZER TO A JOINT CHANGE IN CG POSITION AND AXIAL MOMENT OF INERTIA

QE (deg)	V_o/Zone (f/s)	Modif. No.*	Range (m)	Defl. (m)	Δ Defl. (m)	Correction (mils)
12	1040/Z3	0	3822	11	0	0
		1	3822	11	0	0.1
24		0	6627	42	0	0
		1	6627	41	- 1	0.3
45		0	8547	136	0	0
		1	8548	130	- 6	0.7
12	1960/Z7	0	9399	55	0	0
		1	9399	53	- 2	0.2
24		0	13967	158	0	0
		1	13966	152	- 6	0.4
45		0	17622	398	0	0
		1	17622	382	- 16	1.0
12	2440/Z9	0	12869	84	0	0
		1	12869	81	- 3	0.2
24		0	18369	247	0	0
		1	18368	237	- 10	0.5
45		0	22853	621	0	0
		1	22853	595	- 26	1.2

* For modification number n:

n	Loc. of CG (cal re nose)	Axial MI ($kg\ m^2$)
0	3.592	0.570
1	3.577	0.547

TABLE 2.4A. CHARACTERISTICS OF THE STANDARD PROJECTILE

caliber	203.2 mm
mass	200 lb
cg position	2.5 cal aft of nose
axial moment of inertia	0.5 slug ft^2 (0.6779 kg m^2)
pitch moment of inertia	4.0 slug ft^2 (5.4234 km m^2)
center of pressure	1.73 cal at Mach 1.77
projectile length	4.3 cal (34.4 in)
muzzle velocity	1950 f/s at zone 7 in M2 howitzer
initial spin	735 rad sec^{-1}

TABLE 2.4B. DRAG COEFFICIENT FOR THE STANDARD PROJECTILE

Mach No.	C_D (form)	C_D (skin)	C_D (total)
0.0	0.126	0.056	0.182
0.8	0.126	0.049	0.175
0.9	0.190	0.048	0.238
1.0	0.302	0.047	0.349
1.1	0.307	0.046	0.353
1.2	0.300	0.046	0.346
1.5	0.262	0.045	0.307
2.0	0.210	0.043	0.253

TABLE 2.5A. CHARACTERISTICS OF THE M106 PROJECTILE FIRED FROM THE M2A2 CANNON

caliber	203.2 mm
mass	200 lb
cg position re nose	2.840 cal
projectile length	4.375 cal
axial moment of inertia	0.553 kg m^2
pitch moment of inertia	4.270 kg m^2
muzzle velocity	1950 f/s at zone 7
	1380 f/s at zone 5
initial spin	735 rad/s at zone 7
	520 rad/s at zone 5

TABLE 2.5B. DRAG COEFFICIENT* FOR THE M106 PROJECTILE

Mach No.	C_D
0.00	0.125
0.75	0.125
0.85	0.129
0.90	0.140
0.95	0.152
1.00	0.351
1.05	0.400
1.10	0.400
1.50	0.356
2.00	0.305
2.50	0.280

* BRL estimate [6]

[6] Dubin, J.A., et al. Ballistic Similitude: 8 Inch Ammunition, (SECRET), Technical Report 4165, Picatinny Arsenal, Dover, N.J., June, 1973.

TABLE 2.6A. CHARACTERISTICS OF THE M509 ICM PROJECTILE USED WITH THE M2A2 AND THE XM201 CANNONS

caliber	203.2 mm
mass	205.9 lb
cg position re nose	3.592 cal
projectile length	5.674 cal
axial moment of inertia	0.570 kg m^2
pitch moment of inertia	4.7676 kg m^2
muzzle velocity (max)	1906 f/s in M2A2
	2240 f/s in XM201
initial spin	718.5 rad/s in M2A2
	1150 rad/s in XM201

TABLE 2.6B. DRAG COEFFICIENT* FOR THE M509 PROJECTILE

Mach No.	C_D
0.00	0.130
0.75	0.130
0.85	0.140
0.90	0.155
1.00	0.300
1.05	0.360
1.10	0.360
1.50	0.317
2.00	0.274
2.50	0.239

* BRL estimate [6]

[6] Dubin, J.A., et al. Op. Cit.

CHAPTER III

VIBRATION OF THE LIQUID IN A SPINNING LIQUID-FILLED PROJECTILE

Introduction

In an effort to assess the consequences of vibration of the liquid surface while in Configuration B on the flight stability of the projectile, a simple methodology is proposed. First, one must examine the admissible shapes which the liquid surface can assume. For a set of vibrational modes of interest one can then estimate the associated natural vibrational frequencies for the liquid, treated as a conservative system. To be analytically tractable this treatment will assume the liquid to be rotating at a constant angular velocity ω. Actually, of course, the liquid does not have a constant ω everywhere during spinup and, further, at any point in the liquid ω depends upon time. Therefore, the degree of credibility of results derived from the above assumption will depend upon the relative magnitude of liquid acceleration due to spinup and the centrifugal acceleration due to spin. If the latter is much larger than the former, it is plausible to treat liquid vibration pseudostatically.

The ultimate goal of the present analysis is to compare the natural vibrational frequencies of the liquid with the frequencies of precession and nutation of the entire projectile. If any of the vibrational frequencies were found to remain close to the precessional or nutational frequency of the projectile during flight, a resonant condition could occur in which the system vibrational modes excited each other at their common frequency. Conceivably this could cause projectile flight instability if the liquid vibration was severe enough. At the very least a "mode lock" of this sort would increase projectile dispersion.

To motivate further developments, one should note that the precessional and nutational frequencies of concern are rather low, lying in the band from 0 to 20 hertz, approximately. For stable projectiles such as the M106 or the M509, the precessional frequency is typically about 1 to 3 hertz thruout flight. The Appendix includes derivations of the equations for precessional frequency and nutational frequency of a spin-stabilized projectile.

By contrast to the very low precessional frequency, nutational or yawing frequency is of greater concern relative to projectile stability* in liquid-filled projectiles since this frequency is such that a liquid vibrational frequency will cross it during spinup. To illustrate the band of nutational frequencies, Figure 3.1 displays the nutational frequency versus time, for several firing zones, during the flight of the M509 projectile from the XM201 cannon. Figure 3.2 shows a similar result for the M106 projectile from the M2A2 cannon.

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* A stable projectile is one in which pitching or yawing motions are ultimately reduced in amplitude during flight without the projectile being completely overturned. In practice a condition of neutral, dynamic stability, such that amplitudes remain constant, is difficult to obtain. If the system starts to progressively increase its yaw, it does so quickly and ultimately overturns.

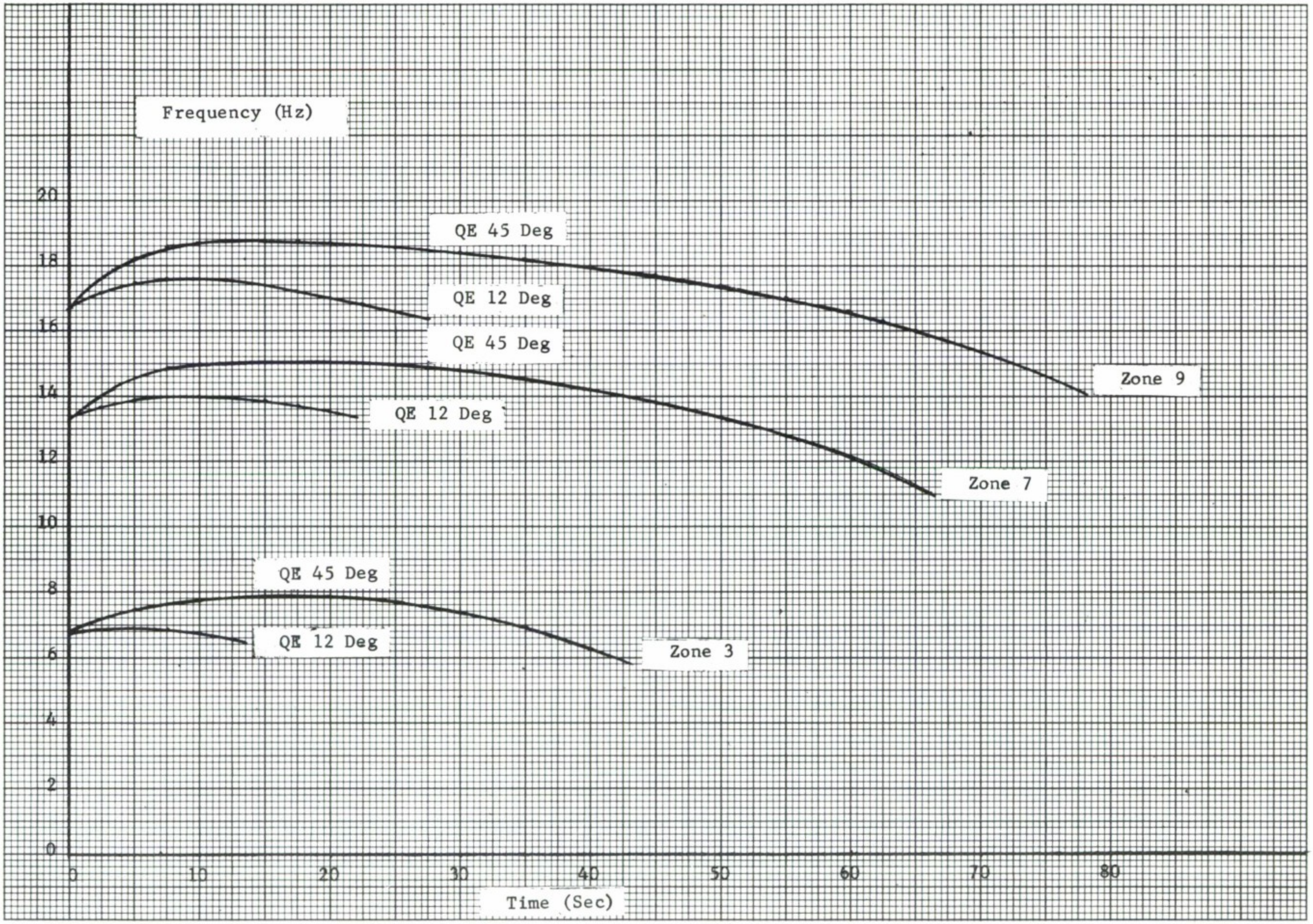

Figure 3.1. Nutational Frequency of the M509 Projectile at Several Zones in the XM201 Cannon

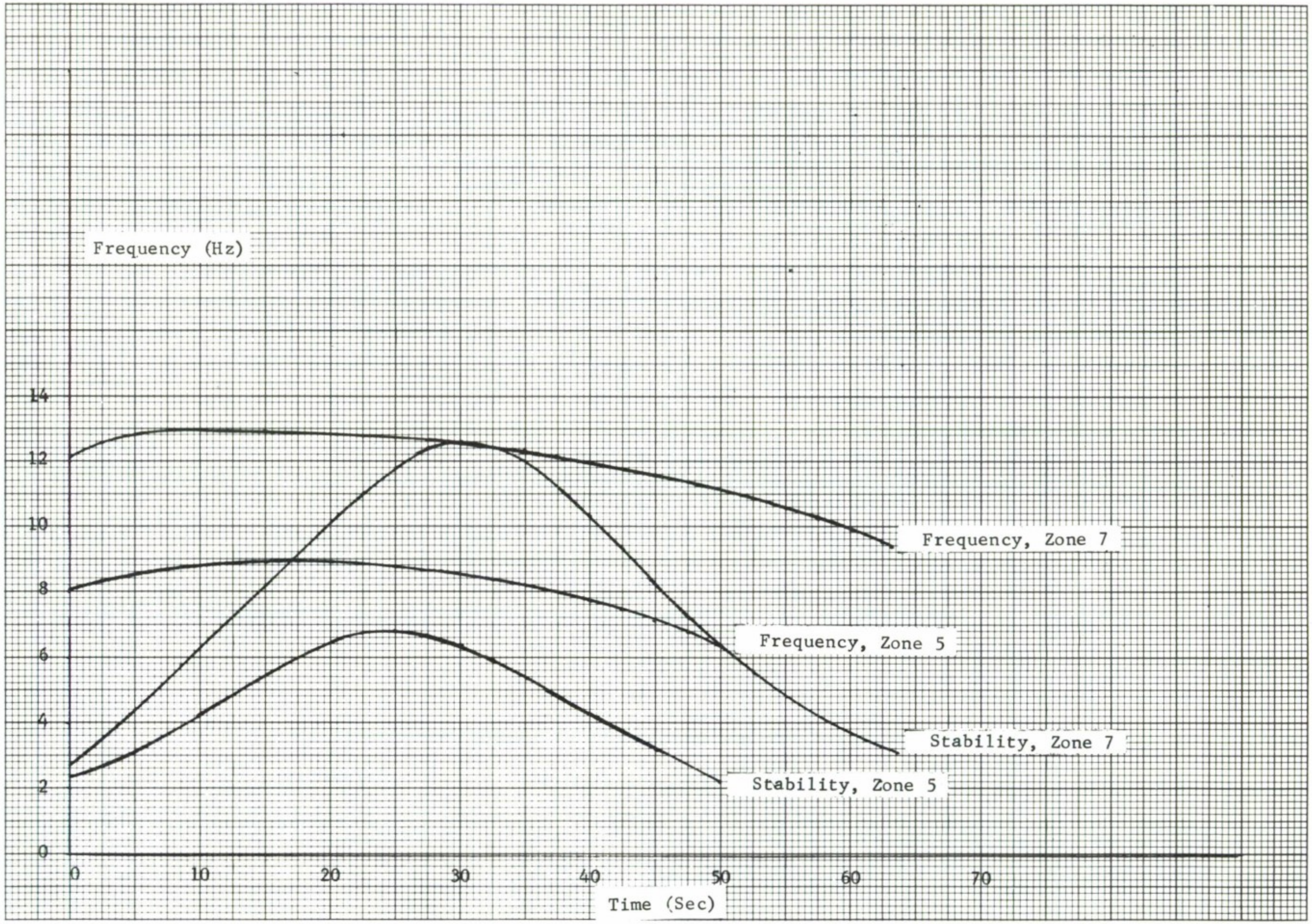

Figure 3.2. Nutational Frequency and Stability Factor During Flight of the M106 Projectile from the M2A2 Cannon

Vibrational Modes

We proceed with the analysis outlined above by considering perturbations to the liquid surface of Configuration B. In Figure 3.3 below, the equilibrium position of the free surface of the liquid at its inner radius is designated y_o. The perturbation or deflection of this surface is δ where

$$\delta = f(x) \quad . \tag{3.1}$$

Thus the position of the surface, y, is given by

$$y = y_o + \delta \quad . \tag{3.2}$$

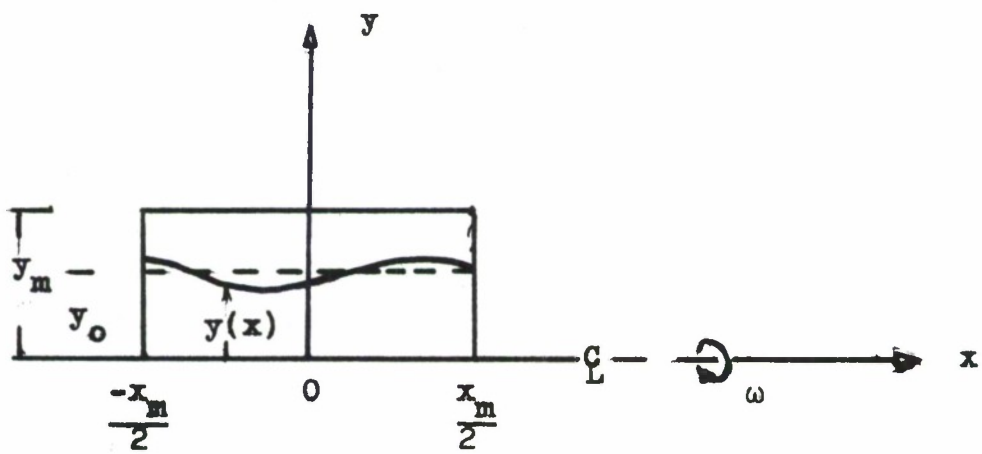

Figure 3.3. Surface of the Vibrating Liquid

The liquid volume v is given by

$$v = -\pi \int_{-x_m/2}^{x_m/2} y^2 \, dx + \pi \, y_m^2 \, x_m \tag{3.3}$$

$$v = \pi\, x_m(y_m^2 - y_o^2) \tag{3.4}$$

Then with a constant liquid volume given by (3.4) and with δ small, i.e.,

$$\delta << y_o,$$

$$\int_{-x_m/2}^{x_m/2} f(x)\, dx \cong 0 \quad . \tag{3.5}$$

Equation (3.5) places a constraint on the form of f(x).

Now expand f(x) in a Fourier series, i.e.,

$$f(x) = \sum_{n=1}^{\infty} a_n \cos \frac{2\,\pi\,n\,x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{2\,\pi\,n\,x}{L} \quad . \tag{3.6}$$

Furthermore since we are interested in vibrational modes which might amplify yaw and destabilize the projectile, consider only odd components of f(x) for the present. In this special case

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{2\,\pi\,n\,x}{L} \quad . \tag{3.7}$$

Applying the constraint given by (3.5),

$$\int_{-x_m/2}^{x_m/2} dx \sum_{n=1}^{\infty} b_n \sin \frac{2\,\pi\,n\,x}{L} = 0 \quad .$$

After exchanging the order of operations

$$\sum_{n=1}^{\infty} b_n \int_{-x_m/2}^{x_m/2} \sin \frac{2 \pi n x}{L} = 0$$

or

$$2 \sum_{n=1}^{\infty} b_n \cos \frac{\pi n x_m}{L} = 0 \quad , \tag{3.8}$$

for all integer n. For this expression to hold for an arbitrary set b_n, the argument of the cosine must be an odd integer multiple of $\pi/2$.

$$\frac{\pi n x_m}{L} = (2 k - 1) \frac{\pi}{2} \quad , \quad 1 \leq k$$
$$, \quad 1 \leq n$$

Therefore,

$$L = \frac{2 n x_m}{2 k - 1} \tag{3.9}$$

and with (3.7)

$$f(x) = \sum_{k=1}^{\infty} b_k \sin \frac{\pi (2 k - 1) x}{x_m} \quad . \tag{3.10}$$

Since we are principally interested in low frequency vibrations, we restrict the following development to the fundamental mode, i.e., to k = 1. Then

$$f(x) = b \sin \frac{\pi x}{x_m} \quad , \quad \frac{-x_m}{2} < x < \frac{x_m}{2} \quad . \tag{3.11}$$

The Fourier amplitude b will serve as the single degree of freedom characterizing the vibrations of the liquid surface.

Energy Considerations

At this point a brief excursion in the development is necessary to discuss the pressure in, and energy of, the rotating liquid.

The rotational kinetic energy of the liquid is

$$T_{rot} = \pi \rho \int_{-x_m/2}^{x_m/2} dx \int_{y}^{y_m} r^3 \omega^2 dr \qquad (3.12)$$

where liquid density is ρ and where the angular velocity ω may depend upon both x and r. At dynamic equilibrium

$$y = y_o \; ,$$

$$\omega = \omega_o \quad \text{and}$$

$$T_{rot} = \frac{\pi}{4} \rho \, \omega_o^2 \, x_m \, y_m^4 \, (1 - q^4) \qquad (3.13)$$

with

$$q = y_o/y_m \; .$$

The potential energy of the liquid in an equilibrium configuration is due only to the compression of the liquid under centrifugal force. The compression of the differential volume element dv,

$$dv = 2 \pi r \, dr \, dx \quad , \qquad (3.14)$$

is due to the pressure needed to support the liquid below r, i.e., at a radius smaller than r. In a liquid rotating as a solid body with constant angular velocity ω_o

$$\frac{dp}{dr} = \rho\, \omega_o^2\, r \tag{3.15}$$

which upon integration yields

$$p(r) = \frac{\omega_o^2 \rho}{2} (r^2 - y_o^2) \quad , \quad y_o \leq r \leq y_m \quad . \tag{3.16}$$

If k is the volumetric compressibility of the liquid, i.e.,

$$k = \frac{1}{v} \frac{\partial v}{\partial p} \quad ,$$

then the compressional potential energy associated with the differential volume dv is

$$d\, V_c = \frac{k}{2} p^2\, dv \quad . \tag{3.17}$$

Typically k is 10^{-10} cm² dyne^{-1} for liquids.

The total compressional potential energy

$$V_c = \int_v \frac{k}{2} p^2\, dv \quad .$$

And, (3.14, 3.17) yield

$$V_c = \pi\, k \int_{-x_m/2}^{x_m/2} dx \int_y^{y_m} p^2\, r\, dr \quad . \tag{3.18}$$

For a liquid in dynamic equilibrium, p(r) is given by (3.16). Using this result

$$V_c = \pi\ k\ x_m \int_{y_o}^{y_m} \frac{\omega_o^4\ \rho^2}{4}\ (r^2 - y_o^2)^2\ r\ dr$$

or

$$V_c = \frac{\pi}{24}\ \omega_o^4\ \rho^2\ k\ x_m\ y_m^6\ (1 - q^2)^3$$

with

$$q = y_o/y_m\ . \qquad (3.19)$$

To appreciate the significance of the value of liquid pressure, the rotational kinetic energy, and the compressive potential energy, we present the following numerical example.

Example

Using values used in previous examples,

$$\omega_o = 942\ \text{rad sec}^{-1}$$

$$\rho = 1\ \text{gm cm}^{-3}$$

$$x_m = 55.88\ \text{cm (22 in)}$$

$$y_m = 7.62\ \text{cm (3 in)}$$

$$y_o = 2.516\ \text{cm (0.991 in)}$$

and

$$k = 10^{-10}\ \mathrm{cm}^2\ \mathrm{dyne}^{-1}$$

$$q = 0.3302 \quad .$$

From (3.16), the pressure at radial position y_m is given by

$$p(y_m) = \frac{\omega_o^2\ \rho\ y_m^2}{2} (1 - q^2)$$

$$= \frac{(942)^2 (7.62)^2}{2} (1 - 0.3302^2)$$

$$p(y_m) = 2.295\ 10^7\ \mathrm{dyne\ cm}^{-2}$$

$$= 22.65\ \mathrm{atm}$$

$$= 332.95\ \mathrm{psi}$$

And from (3.19),

$$V_c = \frac{\pi}{24} (942)^2\ (10^{-10})\ (55.88)\ (7.62)^6\ (1 - 0.3302^2)^3$$

$$V_c = 7.973\ 10^7\ \mathrm{dyne\ cm} \quad \text{or } 7.973 \text{ joules} \quad .$$

Finally, from (3.13)

$$T_{rot} = \frac{\pi}{4} (942)^2\ (55.88)\ (7.62)^4\ (1 - 0.3302^4)$$

$$T_{rot} = 1.2974\ 10^{11}\ \mathrm{dyne\ cm}$$

$$T_{rot} = 12{,}974 \text{ joules} \quad \text{or} \quad 9{,}569\ \mathrm{ft\ lb_f} \quad .$$

The potential energy of the liquid at equilibrium is only 0.061% of the rotational kinetic energy under this condition. Therefore one would not expect the exchange of energy between kinetic and compressional potential forms to contribute significantly to liquid vibrations. Compressional potential energy is assumed negligible in subsequent calculations.

Estimate of a Fundamental Vibrational Frequency

At this point we return to the principal arguments associated with the derivation of an expression for the vibrational frequency of the liquid surface. To develop this expression, an estimate of the vibrational kinetic energy of the liquid will be required.

An estimate of the kinetic energy associated with a longitudinal vibrational mode of the liquid can be made simply by making these assumptions:

(1) The liquid surface during vibration remains axisymmetric; i.e., circumferential modes are not excited.

(2) The functional form describing the liquid surface changes with time only thru time-dependence of the coefficients in a Fourier transform of the function, i.e., only thru the Fourier amplitudes.

(3) The column length for flow of a liquid element at position x (relative to the center of the disturbance) is proportional to x.

By the first and second assumptions the first odd vibrational mode --

$$y = b \sin \frac{\pi x}{x_m}$$

has time derivative

$$\dot{y} = \dot{b} \sin \frac{\pi x}{x_m} . \tag{3.20}$$

At the surface element

$$2 \pi y_o dx ,$$

the path or column length of a control volume within which this surface element vibrates is

$$c x , \quad 0 < x < \frac{x_m}{2} , \tag{3.21}$$

by the third assumption, where c is a constant. This constant can be evaluated by requiring that the volume integral of all elemental control volumes produces the volume of the liquid v_ℓ. That is

$$v_\ell = \int_o^{x_m/2} 2 \pi y_o c x dx$$

or

$$c = \frac{4 v_\ell}{\pi y_o x_m^2} . \tag{3.22}$$

But

$$v_\ell = \pi y_m^2 x_o , \tag{3.23}$$

where x_o is the length of the liquid cylinder in Configuration A.

Then

$$c = \frac{4 x_o y_m^2}{x_m^2 y_o} . \tag{3.24}$$

In previous examples

$x_o = 19.6$ in

$x_m = 22$ in

$y_o = 0.991$ in

$y_m = 3$ in

so that

$c = 1.471$.

The kinetic energy of vibration of an elemental volume is

$$\pi\ c\ \rho\ y_o\ x\ dx\ \dot{y}^2 \quad , \quad 0 < x \leq \frac{x_m}{2} \quad .$$

The total vibrational kinetic energy is obtained by volume integration of this expression with $\dot{y}$ given by (3.20).

$$T_{vib} = \pi\ c\ \rho\ y_o\ \dot{b}^2 \int_o^{x_m/2} x \sin^2 \frac{\pi x}{x_m}\, dx \tag{3.25}$$

$$T_{vib} = \frac{(\pi^2 + 4)}{16\ \pi}\ c\ \rho\ y_o\ x_m^2\ \dot{b}^2 \tag{3.26}$$

or

$$T_{vib} = K_t\ \dot{b}^2$$

with

$$K_t = \frac{(\pi^2 + 4)}{16\ \pi}\ c\ \rho\ y_o\ x_m^2 \quad . \tag{3.27}$$

Thus the vibrational kinetic energy is simply proportional to the square of the time derivative of the Fourier amplitude.

An expression for the vibrational potential energy associated with this mode is developed as follows. Let $p(r,y_o)$ be the pressure in the liquid at radial position r when the liquid is in Configuration B. An expression for $p(r,y_o)$ is given in (3.16). Then, providing the displacement δ of the surface from y_o is small, the work performed to effect a change in liquid configuration from the equilibrium position y_o to a terminal position y is

$$W = \int_{x=o}^{x_m/2} dx \int_{z=o}^{b} 2\pi y_o \left[p\,(y_o + \delta,\ y_o - \delta)\ d\delta\right]$$

with

$$\delta = z \sin \frac{\pi x}{x_m} \quad . \tag{3.28}$$

$$W = \frac{\pi}{2} \rho\, \omega_o^2\, y_o^2\, x_m\, b^2 \tag{3.29}$$

But the vibrational potential energy is equal to the work done to change the configuration. Thus

$$V_{vib} = K_v\, b^2$$

with

$$K_v = \frac{\pi}{2} \rho\, \omega_o^2\, y_o^2\, x_m \quad . \tag{3.30}$$

For b equal to 1 cm and the other parameters having the values given in the previous example, V_{vib} equals 49.3 joules or only 0.38% of the rotational kinetic energy.

Having obtained expressions for the potential and kinetic energy of vibration of the liquid in terms of the Fourier amplitude b and its first derivative, one can obtain the equation of motion of the liquid surface by direct application of Lagrange's equation for a conservative system.

$$\frac{d}{dt}\frac{\partial T_{vib}}{\partial \dot{b}} + \frac{\partial V_{vib}}{\partial b} = 0 \tag{3.31}$$

This result does, of course, neglect the effects of dissipative forces.

But from (3.27)

$$\frac{\partial T_{vib}}{\partial \dot{b}} = 2\,K_t\,\dot{b}$$

and from (3.30)

$$\frac{\partial V_{vib}}{\partial b} = 2\,K_v\,b \quad .$$

Therefore,

$$\ddot{b} + \frac{K_v}{K_t}\,b = 0 \quad . \tag{3.32}$$

The undamped angular frequency associated with this vibrational mode by inspection of (3.32) is

$$\Omega_{vib} = (K_v/K_t)^{\frac{1}{2}} \tag{3.33}$$

$$\frac{\Omega_{vib}}{\omega_o} = \left[\frac{8\pi^2 y_o}{(\pi^2 + 4) c x_m}\right]^{\frac{1}{2}} \tag{3.34}$$

Since the natural vibratory frequency ν_{vib} is $\Omega_{vib}/2\pi$,

$$\nu_{vib} = \left[\frac{2 y_o}{(\pi^2 + 4) c x_m}\right]^{\frac{1}{2}} \omega_o \quad . \tag{3.35}$$

With the parameter values used in previous examples,

$y_o = 0.991$ in

$x_m = 22$ in

$c = 1.471$,

$$\nu_{vib} = 0.06645 \, \omega_o \quad . \tag{3.36}$$

The value of ω_o in this expression is interpreted as the effective angular velocity of the liquid, i.e., that uniform angular velocity which produces the observed angular momentum when ω is not uniform thruout. At the muzzle spin for zone 7 of the M2 howitzer

$\omega_o = 735$ rad sec^{-1}

and

$\nu_{vib} = 48.8$ hz .

For zone 7 in the M110E2 howitzer

$$\omega_o = 923 \text{ rad sec}^{-1}$$

and

$$\nu_{vib} = 61.3 \text{ hz} \quad .$$

Using (3.36) and the time-dependent, effective angular velocity for the XM736 projectile shown in Figure 2.9, the value of the function $\nu_{vib}(t)$ for this system has been computed. This result is displayed in Figure 3.4. Also shown here for comparison is the nutational frequency at zone 7 for the XM736 projectile.

It is noted that the spinup of a liquid with constant kinematic viscosity of 10 centistokes occurs rapidly enough to cause the vibrational frequency to quickly cross over the nutational frequency. In this case liquid-vibration-induced projectile instability is unlikely. With a liquid having lower viscosity while in Configuration B, the crossover of frequencies is not so abrupt and may at least produce additional dispersion. Considering the magnitude of the perturbing effect of liquid vibration, complete projectile instability is unlikely.

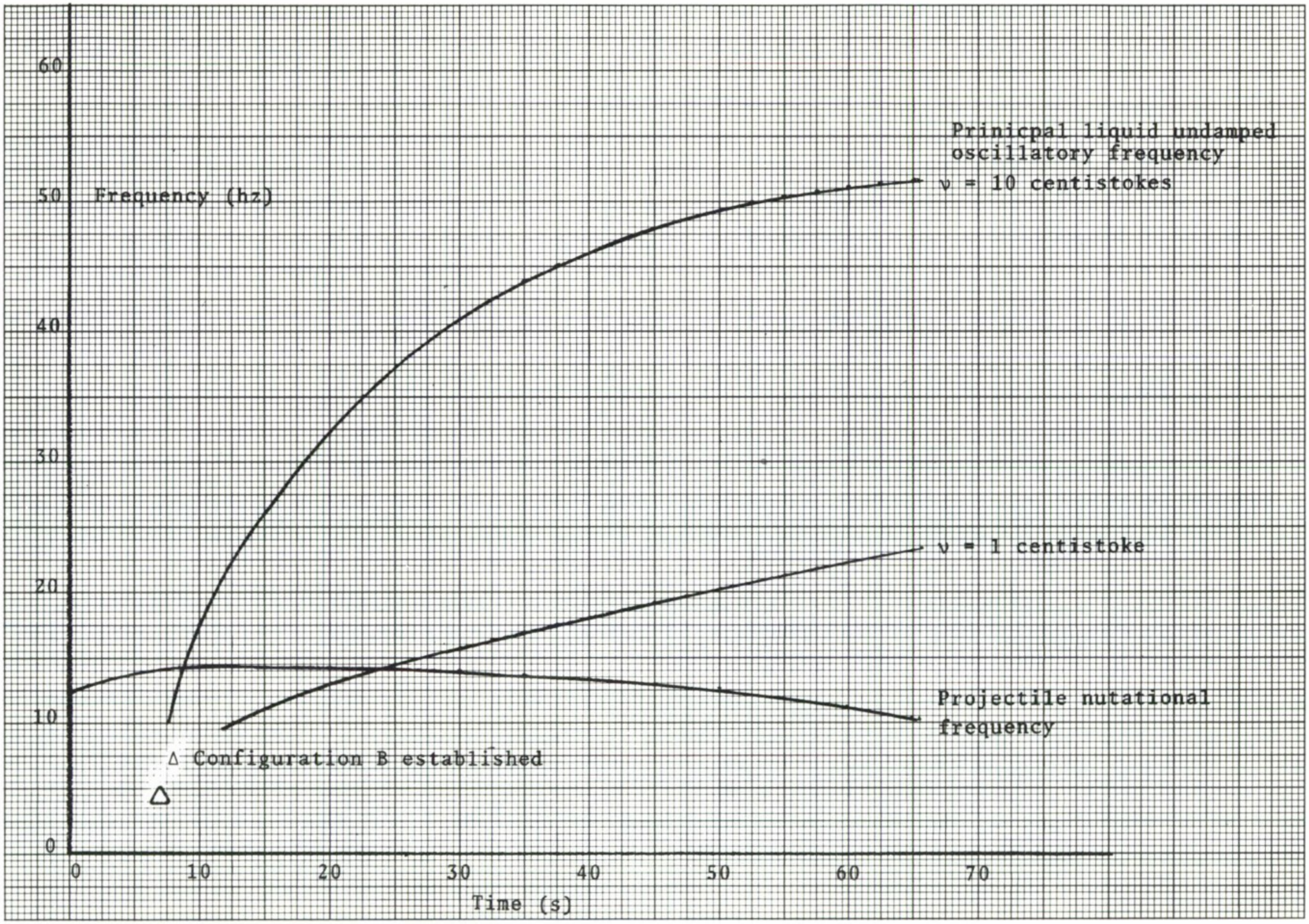

Figure 3.4. Comparison of Oscillatory Modes of the XM736 Liquid-Filled Projectile (Zone 7 at QE 45^{o} in M110E2 Howitzer)

In view of the approximations required to perform this type of analysis, stronger conclusions are not warranted.* An experimental program to better define the values of the pertinent parameters characterizing the liquid certainly is suggested. Additionally, a program to define the ballistic dispersion of projectiles with low-viscosity liquid fills is indicated.

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*Note [3.1]:

A more elaborate analysis such as a finite-element description of the liquid, embedded in a six-degree-of-freedom flight simulation appears to be a profitable direction to take analytically; however, this method will likely blur some of the insights afforded by the simpler procedure used here.

Some of the restrictive assumptions used in our analysis could be removed. As an example we note that our analysis in Chapter III neglects certain forces arising because of the non-Newtonian character of the coordinate system. A coordinate system which rotates with the projectile is assumed here. Since this is a non-Newtonian frame, a modified form of Euler's equations must be used. This formulation introduces the usual centrifugal and Coriolis forces as well as an "angular acceleration" force which arises from the angular acceleration of the projectile. The latter force is in the same direction as the Coriolis force and is proportional to the angular acceleration of the projectile.

For a liquid element of mass m in this rotating coordinate system, the apparent acceleration vector $\underline{a}_m$ in the moving frame due to an external force F is given by the equation

Note [3.1] (continued)

$$\underline{a}_m = \underline{F}/m - \ddot{\underline{r}}_o - 2\,\underline{\omega} \times \dot{\underline{r}}_{ma} - \dot{\underline{\omega}} \times \underline{r}_m - \underline{\omega} \times (\underline{\omega} \times \underline{r}_m) \quad ,$$

where

$\underline{r}_o$ is the origin of the moving frame relative to a Newtonian frame

$\underline{r}_m$ is the position of the mass element in the moving frame

$\dot{\underline{r}}_{ma}$ is the apparent velocity of the mass element in the moving frame

$\underline{\omega}$ is the angular velocity of the moving frame in a Newtonian frame

$\dot{\underline{\omega}}$ is the angular acceleration of the moving frame (projectile)

The term $- \underline{\omega} \times (\underline{\omega} \times \underline{r}_m)$ is the centrifugal acceleration which has been treated. The term $- 2\,\underline{\omega} \times \dot{\underline{r}}_{ma}$ is the Coriolis acceleration. In a right-handed frame with the x-axis along the spin axis of the projectile, this term reduces to

$$- 2\,\omega\,(\dot{y}\,\underline{k} - \dot{z}\,\underline{j})$$

with $\underline{j}$ and $\underline{k}$ unit vectors in the y- and z-directions. If the transverse components of velocity are small, this term is negligible and, in fact, was neglected. The term $- \dot{\underline{\omega}} \times \underline{r}_m$ is the angular acceleration component of acceleration and reduces to

$$- \underline{k}\,\dot{\omega}\,y + \underline{j}\,\dot{\omega}\,\dot{z} \quad .$$

This term has also been neglected due to the small value of $\dot{\omega}$.

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APPENDIX

DERIVATIONS AND COMPUTER PROGRAMS

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APPENDIX

DERIVATIONS AND COMPUTER PROGRAMS

Estimate of Precessional and Nutational Frequencies of a Spin-Stabilized Projectile

For a gyroscopically stable spinning system with angular velocity ω and axial moment of inertia I_A, the precessional angular velocity Ω obtained when the gyro feels a torque τ is given by

$$\Omega = \frac{\tau}{I_A \omega} \quad . \tag{A1}$$

If the torque is produced by aerodynamic forces where the center of pressure is $- X_{sm}$ calibers in front of the cg, then

$$\tau = - X_{sm} D C_N A q = - X_{sm} D C_{N_\alpha} \alpha A q \quad . \tag{A2a}$$

where, notationally and by definition,

D = caliber

C_N = normal force coefficient

C_{N_α} = normal force derivative coefficient

α = projectile angle of attack

A = reference area

$$A = \frac{\pi}{4} D^2 \tag{A2b}$$

q = dynamic pressure

$$q = \frac{\rho V^2}{2} \quad . \tag{A2c}$$

Since the overturning-moment coefficient

$$C_M = - X_{sm} C_N = (X_{cg}-X_{cp}) C_N \quad , \tag{A3}$$

equation (A2a) may be written

$$\tau = C_M D A q$$

$$\tau = C_{M_\alpha} \alpha D A q \quad . \tag{A4}$$

The natural precessional frequency at constant α is

$$\nu_p = \frac{\Omega}{2\pi}$$

$$\nu_p = \frac{C_M D A q}{2\pi I_A \omega} \quad . \tag{A5}$$

Yaw Frequency

The solution of the linearized equations for the yawing motion of a projectile is given by McShane et al on p. 644 of [7]. The solution involves superposition of two sinusoids having different frequencies. The two frequencies are

$$\Omega_{1,2} = \frac{I_A \omega}{2 I_B} (1 \pm \sigma) \qquad \text{(rad/s)}$$

or

$$\nu_{1,2} = \frac{1}{2\pi} \Omega_{1,2} \qquad \text{(hz)} \tag{A6}$$

with

$$\sigma = (1-s^{-1})^{\frac{1}{2}} \quad , \tag{A7}$$

where s is the gyroscopic stability factor given by

$$s = \frac{I_A^2 \omega^2}{8 I_B K_{M_\alpha} D^3 q}$$

[7] McShane, E., Kelly, J. and Reno, F. Exterior Ballistics, University of Denver Press, c. 1953.

or

$$s = \frac{I_A^2 \, \omega^2}{\pi \, I_B \, C_{M_\alpha} \, D^3 \, q} \quad . \tag{A8}$$

The frequencies $\nu_{1,2}$ have been called the "nutational" and "precessional" frequency, respectively. However, it should be noted that ν_1 is not the reciprocal of the time between yaw maxima. Because of the superposition of the high- and low-frequency components of motion, the frequency with which maximum yaw occurs is the difference frequency:

$$\nu_y = \nu_1 - \nu_2 \quad . \tag{A9}$$

This is called the yaw or nutational frequency in this report. An alternative derivation for ν_y starts with an expression for the wavelength of yaw Λ, in calibers. Then,

$$\nu_y = \frac{V}{D \, \Lambda} \qquad \text{(hz)} \tag{A10}$$

with projectile velocity V and caliber D.

An expression for Λ is obtained from p. 651 of [7]. Using our notation

$$\Lambda = \frac{2 \, \pi \, I_B \, V}{I_A \, D \, \omega \, \sigma} \quad . \tag{A11}$$

Then, (A10) and (A11) yield

$$\nu_y = \frac{\omega}{2 \, \pi} \, \frac{I_A \, \sigma}{I_B} \quad . \tag{A12}$$

This frequency has been computed as a function of time for several combinations of projectiles and cannons. Results for the M509 projectile in the XM201 cannon are shown in Figure 3.1. Comparable results for the M106 projectile in the M2A2 cannon are shown in Figure 3.2.

[7] McShane, E., Kelly, J. and Reno, F. Op. Cit.

Derivation of an Equation for Spin Decay in Projectiles

Glossary of Terms

N = projectile spin (rad/sec)

N_o = initial spin (rad/sec)

V = projectile velocity (m/sec)

V_o = muzzle velocity (m/sec)

t = time since launch (sec)

ρ = air density (kg/m^3)

D = projectile caliber (m)

M = projectile mass (kg)

I_A = projectile axial moment of inertia (kg m^2)

K_A = spin damping moment coefficient

K_D = zero-lift drag coefficient

k = velocity decay parameter (m^{-1})

x = rangewise coordinate (m)

$$\dot{N} = -\frac{\rho D^4}{I_A} K_A N V \tag{A13}$$

For flat trajectories at low QE, we assume negligible gravitational effects.

$$\dot{V} = -\frac{K_D \rho D^2}{M} V^2 \tag{A14}$$

$$\frac{d(V)^{-1}}{dt} = \frac{K_D \rho D^2}{M} \tag{A15}$$

$$V^{-1} - V_o^{-1} = \frac{K_D \rho D^2}{M} t \tag{A16}$$

$$V = (V_o^{-1} + kt)^{-1} \tag{A17}$$

with $$k = \frac{K_D \rho D^2}{M} . \tag{A18}$$

Then,

$$\dot{N} = - \alpha V N \quad \text{with} \tag{A19}$$

$$\alpha = \frac{\rho D^4 K_A}{I_A} \tag{A20}$$

$$\frac{\dot{N}}{N} = - \frac{\alpha}{V_o^{-1} + kt} \tag{A21}$$

$$d(\ln N) = - \frac{\alpha \, dt}{V_o^{-1} + kt} \tag{A22}$$

$$\ln N \Big|_{N_o}^{N} = - \frac{\alpha}{k} \ln (V_o^{-1} + kt) \Big|_{0}^{t}$$

$$\ln \frac{N}{N_o} = - \frac{\alpha}{k} \ln (1 + k V_o t) \tag{A23}$$

$$\frac{N_o}{N} = (1 + k V_o t)^{\beta} \tag{A24}$$

with $$\beta = \frac{\alpha}{k} = \frac{K_A M D^2}{K_D I_A} \tag{A25}$$

or $$N = \frac{N_o}{(1 + k\,V_o\,t)^{\beta}} \quad . \tag{A26}$$

Integration of (A17) produces

$$V = V_o\, e^{-kx} \tag{A27}$$

And with (A16),

$$t = k^{-1}\,(V^{-1} - V_o^{-1}) \tag{A28}$$

or

$$t = k^{-1}\,(V_o^{-1}\, e^{kx} - V_o^{-1})$$

$$t = (e^{kx} - 1)\,/\,(k\,V_o) \tag{A29}$$

Substitution of (A29) into (A26) yields

$$N = N_o\, e^{-\beta\, kx} \quad . \tag{A30}$$

An Example

For long trajectories the variation of drag coefficient and spin-damping moment coefficient with Mach number renders the above results quite approximate. However, in some instances this approximation may be adequate. In this example analytic results for spin damping versus time are compared with those produced by a computer simulation in which variation with Mach number is considered.

Take the M106, 8 inch HE projectile as fired from the M2 howitzer. The maximum range trajectory will be considered. In this case an average altitude ASL is about 10,000 ft. At this altitude air density is about 0.74 sea level standard. Thus

$$\rho = 0.74\ \rho_o = 0.9065\ \text{kg m}^{-3}$$

$$\rho_o = 1.225\ \text{kg m}^{-3}$$

Other parameter values are

$$N(0) = N_o = 735\ \text{rad sec}^{-1}$$

$$V_o \cong 594.4\ \text{m sec}^{-1}$$

$$C_D \cong 0.30\ \text{(effective)} \quad \text{or}$$

$$K_D \cong 0.1178\ \text{(effective)}$$

$$K_A = 0.006\ \text{rad}^{-1}\ \text{(effective)}$$

$$D = 0.203\ \text{m}$$

$$M = 90.72\ \text{kg}$$

$$I_A = 0.678 \text{ kg m}^2$$

Then

$$k = K_D \rho D^2 M^{-1}$$

$$k = 4.8506 \; 10^{-5} \text{ m}^{-1}$$

and

$$\alpha = \rho D^4 K_A I_A^{-1}$$

$$\alpha = 1.3622 \; 10^{-5} \text{ m}^{-1}$$

$$\beta = \alpha / k$$

$$\beta = 0.2808$$

$$V_o k = 0.02883 \text{ sec}^{-1}$$

From (A26)

$$N = N_o (1 + k V_o t)^{-\beta}$$

$$N = 735 (1 + 0.02883 t)^{-0.2808} \tag{A31}$$

The result in (A31) is plotted in the following graph with selected variables from a simulated trajectory. For some purposes the agreement shown between the analytical estimate and the more exact simulation may be satisfactory.

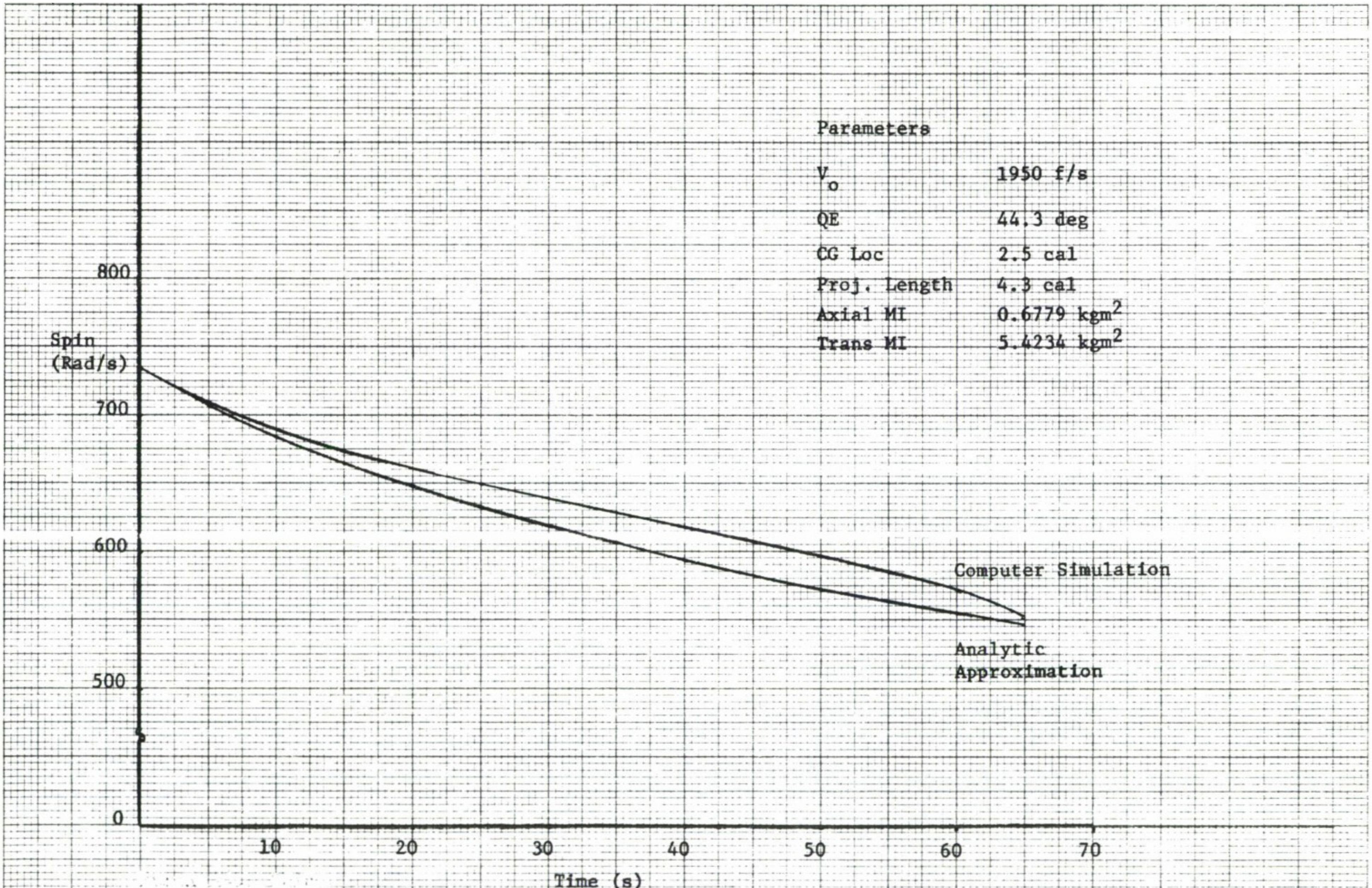

Figure A1. Spin Decay for the Standard Eight-Inch Projectile

Estimate of Stability of Spin-Stabilized Projectiles Having Oscillating Inertial Properties

The objective of this section is to display and illustrate by examples a numerical technique for evaluating the stability of spin-stabilized projectiles which are subject to oscillations in the position of the center of gravity (X_{cg}) and simultaneous oscillations in the transverse moment of inertia (I_y). To preserve the greatest generality of approach, the equations of motion describing projectile pitch-yaw dynamics are solved numerically* in the time domain with cg position and transverse inertia given by the following functions:

$$X_{cg} = X_{cgo} + \delta X_{cg} \sin[2\pi t\,(\nu_o + \dot{\nu}\,t) + \alpha]$$

$$I_y = I_{yo} + \delta I_y \sin[2\pi t\,(\nu_o + \dot{\nu}\,t)] \quad , \qquad \text{(A32)}$$

with X_{cgo}, δX_{cg}, I_{yo}, δI_y, α, ν_o, $\dot{\nu}$ constants for $t \geq 0$.

The solution is carried far enough in time to determine whether the nutational amplitude is decreasing. While admittedly a brute-force approach such as this lacks the elegance of a frequency-domain approach to stability, there is no need to linearize or to assume the amplitudes δX_{cg} and δI_y are small or that the inertial properties oscillate at a constant driving frequency. Further, some insight is gained regarding the behavior of the projectile as the driving frequency matches the nutational frequency of the projectile.

---

* Using a fourth-order Runge-Kutta procedure with time step 0.002 sec.

Using principally the notation of AMCP 706-165, reference [8], the following auxiliary variables in the equations of motion are defined.

$$H = \frac{\rho_a A d}{2 m} [C_{N_\alpha} - 2 C_D - k_y^{-2} (C_{M_q} + C_{M_{\dot\alpha}})]$$

$$M = \frac{\rho_a A d}{2 m} k_y^{-2} C_{M_\alpha} = a^2 A q d C_{M_\alpha} I_y^{-1}$$

$$q = \frac{1}{2} \rho_a v^2$$

$$T = \frac{\rho_a A d}{2 m} [C_{N_\alpha} - C_D + k_x^{-2} C_{M_{p_\alpha}}]$$

$$P = I_x I_y^{-1} \omega a$$

$$a = v^{-1} d$$

$$G = P g_\perp a v^{-1}$$

$$k_x^{-2} = m d^2 I_x^{-1}$$

$$k_y^{-2} = m d^2 I_y^{-1} \quad . \qquad (A33)$$

In this notation:

A = reference area of the projectile

d = caliber of the projectile

m = mass of the projectile

[8] Engineering Design Handbook: Liquid-Filled Projectile Design, AMCP 706-165, April 1969.

I_x = longitudinal moment of inertia

I_y = transverse moment of inertia

k_x = longitudinal radius of gyration in calibers

k_y = transverse radius of gyration in calibers

v = velocity of the projectile

ω = spin of the projectile

ρ_a = air density

$g_{\perp}$ = the component of gravity normal to $\underline{v}$

C_D = the zero-lift drag coefficient

C_{N_α} = the normal force derivative coefficient

C_{M_α} = the overturning moment derivative coefficient

$C_{M_q} + C_{M_{\dot\alpha}}$ = the pitch damping moment coefficient

$C_{M_{p_\alpha}}$ = the Magnus moment derivative coefficient .

Using a right-handed coordinate system as in [8] with the x-axis coinciding with the form axis of the projectile, positive toward the nose, the pitching angular motion in a vertical plane (about the horizontal transverse axis) will be denoted by ϑ and the yaw about the mutually orthogonal axis will be denoted by ψ. With this notation, the equations of motion are:

[8] Op. Cit.

$$\ddot{\vartheta} = a^{-2} M \vartheta - a^{-2} P T \psi - a^{-1} H \dot{\vartheta} - a^{-1} P \dot{\psi} + a^{-2} G$$

$$\ddot{\psi} = a^{-2} P T \vartheta + a^{-2} M \psi + a^{-1} P \dot{\vartheta} - a^{-1} H \dot{\psi} \quad . \qquad (A34)$$

Adopting the systematic notation

$$x_1 = \vartheta$$

$$x_2 = \psi$$

$$x_3 = \dot{\vartheta}$$

$$x_4 = \dot{\psi} \quad , \qquad (A35)$$

equations (A34) become

$$\dot{\underline{x}} = A \, \underline{x} + \underline{b} \qquad (A36)$$

with

$$\underline{x} = [x_1 \; x_2 \; x_3 \; x_4]'$$

$$\underline{b} = [0 \; 0 \; b_3 \; 0]' \quad .$$

The elements of the A matrix, $\{a_{ij}\}$, are given below in units of $\sec^{-2}$.

$$a_{11} = a_{12} = a_{14} = 0$$

$$a_{13} = 1$$

$$a_{21} = a_{22} = a_{23} = 0$$

$$a_{24} = 1$$

$$a_{31} = a^{-2} M = A\, q\, d\, C_{M_\alpha}\, I_y^{-1}$$

$$a_{32} = -\, a^{-2}\, P\, T$$

$$= -\, \omega\, v^{-1}\, d^2\, k_x^2\, A\, q\, I_y^{-1} \quad [C_{N_\alpha} - C_D + k_x^{-2}\, C_{M_{P_\alpha}}]$$

$$a_{33} = -\, a^{-1}\, H$$

$$= -\, A\, q\, v^{-1}\, m^{-1} \quad [C_{N_\alpha} - 2\, C_D - k_y^{-2}\, (C_{M_q} + C_{M_{\dot\alpha}})]$$

$$a_{34} = -\, a^{-1}\, P = -\, I_x\, I_y^{-1}\, \omega$$

$$a_{41} = -\, a_{32}$$

$$a_{42} = a_{31}$$

$$a_{43} = -\, a_{34}$$

$$a_{44} = a_{33}$$

$$b_3 = a^{-2}\, G = I_x\, I_y^{-1}\, g_\perp\, v^{-1} \tag{A37}$$

Examples

Using the equations (A36) and (A37), two projectiles have been treated as numerical examples. Reference [8] provides data on the solid, WP loaded, 152 mm XM410 projectile. Under conditions in which a portion of the white phosphorous fill has liquified, both the effective transverse moment of inertia and center of gravity can be expected to oscillate in flight. Whereas the amplitude of these oscillations may be slight, a persistent resonance of the oscillations with the nutational frequency of the projectile can cause the

nutational amplitude to continuously increase. At a launch Mach number of 1.5, the nominal frequency of yaw maxima for this system is 17.5 hz. Accordingly several numerical experiments were performed in which ν_o (in (A32)) was set to 17.5 and $\dot{\nu}$ was set to 0.0. During these experiments δX_{cg} and δI_y were varied systematically to determine the region of stability. The absolute stability criterion of diminishing yaw amplitude was used here. Preliminary experiments indicated that stability is adversely affected when δX_{cg} and δI_y are in phase. Thus, all experiments were run under the worst-case phase, namely for α in (A32) set to zero.

A second example was selected for comparitive purposes. The numerical values of this example are best estimates of the parameters of the XM736 liquid-filled projectile at a launch Mach number of unity. Axial moment of inertia reflects only that of the metal parts, indicating that the angular momentum of the liquid at launch is treated as negligible. The amplitudes of the inertial increments were selected, somewhat arbitrarily, by taking values proportional to the differences observed in the properties of Configurations A and B of Chapter 1. The values of the parameters used in both examples are displayed in Table A1. Procedures for examining the stability region for the second example were identical to those employed for the first. Stability was examined at a forcing frequency equal to the nominal nutational frequency of 7.77 hz. Additionally, runs were made in which the frequency was swept linearly from 6.0 hz, at the rate of 0.25 hz/sec, to 8.5 hz at 10 sec. These runs clearly showed that, for certain values of δX_{cg} and δI_y, the projectile will remain quite stable under a condition of moving forcing frequency, whereas the projectile

will become unstable when forced at a constant, nutational frequency. The stability region for both examples is displayed in Figure A2. Under the assumed conditions, the projectiles for both examples are stable.

TABLE A1. PARAMETER VALUES FOR GYROSCOPIC STABILITY ANALYSIS

parameter	symbol	value		dimension
		XM410	XM736	
caliber	d	0.5	0.6667	ft
proj. mass	m	1.313	6.31	slug
long. inertia	I_x	0.0446	0.4036	slug ft^2
trans. inertia	I_y	0.1548	3.5164	slug ft^2
amplitude of incr. in trans. inertia	δI_y	0.0006	0.0183	slug ft^2
muzzle velocity	v_o	1675.5	1117	ft s^{-1}
proj. spin	ω	526.4	573.4	rad s^{-1}
air density	ρ_a	2.3769 10^{-3}		slug ft^{-3}
drag coef.	C_D	0.50	0.30	
normal force	C_{N_α}	2.90	2.10	rad^{-1}
pitch damping	$C_{M_q}+C_{M_{\dot\alpha}}$	-5.00	-4.60	(rad sec^{-1})$^{-1}$
magnus moment	$C_{M_{p_\alpha}}$	0.30	-0.10	rad^{-1}
center of press.	X_{cp}	1.40	1.108	cal
proj. center of gravity	X_{cg}	1.85	3.472	cal
amplitude of incr. in c.g.	δX_{cg}	0.04	0.020	cal
gravitational component	$g_\perp$	0.0	0.0	ft s^{-2}

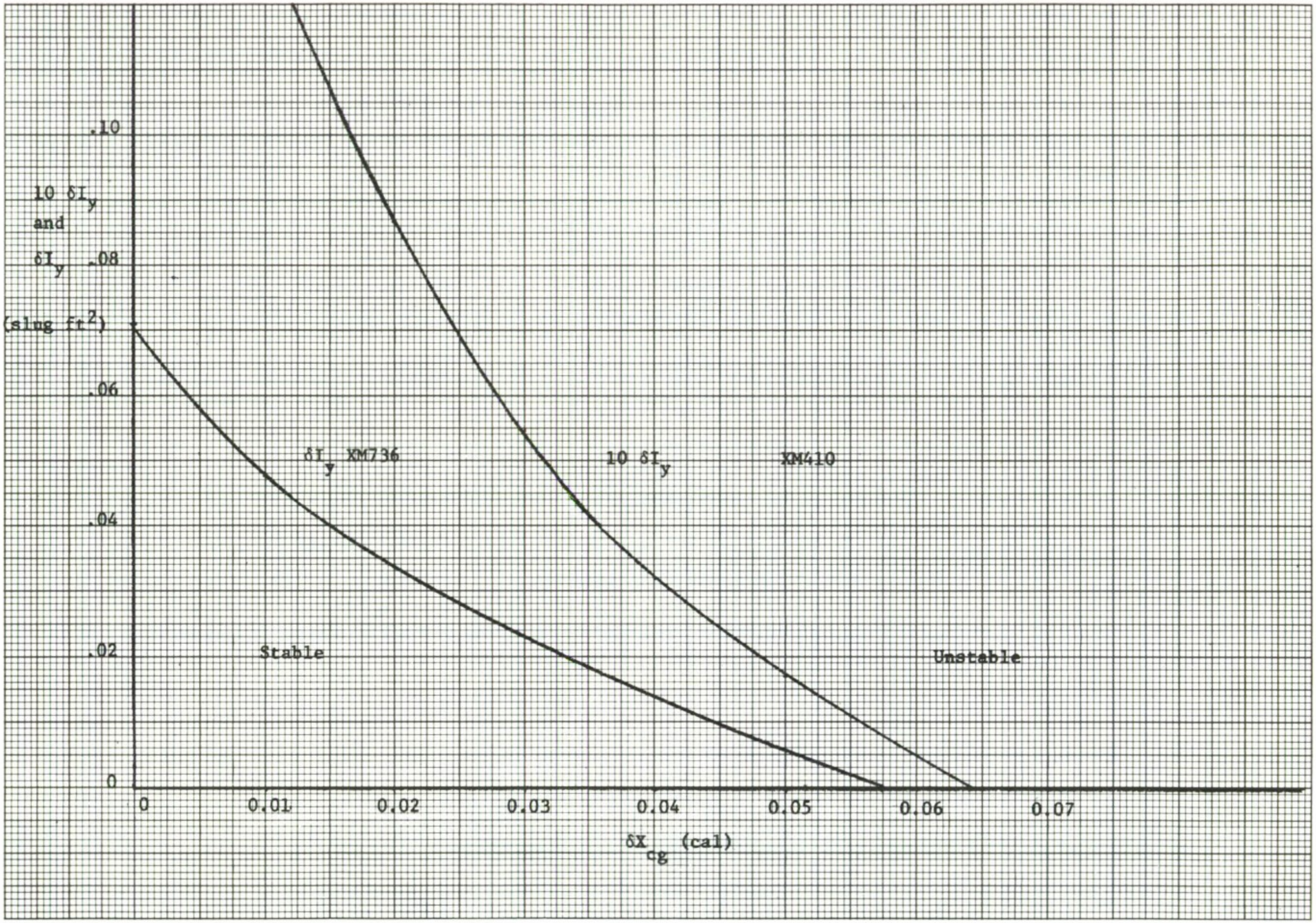

Figure A2. Stability Regions for Projectiles in Examples 1 and 2.

Computer Program for Numerical Solution of One-Dimensional Equations for Spinup of Liquid

The source programs shown on the following pages were written in the FORTRAN IV language for the IBM 360-65 computer. Comments in the listings introduce each main program and subprogram describing its function and delineating the principal operations and variables.

Following each program is a sample of the output produced by the program.

Next page is blank.

```
$JOB   'MGEORGE',KP=29,LINES=560,TIME=360,PAGES=500
C*** DYNAMICS OF SPIN UP OF LIQUID-FILLED PROJECTILE
C
      DIMENSION TITLE(20),U(22),WP(88),X(11)
      COMMON AMBDA,N,X,DX,C1,C2,TO
C*** READ TITLE AND INPUT PARAMS
      READ (5,100) TITLE,DT,TO,TCON,DX,XO,WLIM,N,NPRINT
  100 FORMAT(20A4/6F10.0,2I2)
      WRITE (6,500) N,TCON
  500 FORMAT(1H0,3HN =,I3,2X,6HTCON =,F10.2)
C*** PRINT COLUMN HEADINGS
      WRITE (6,200) TITLE,DT,DX,XO,WLIM.
  200 FORMAT(1H1,20A4/1H011HTIME STEP =,E15.5,14H  SPACE STEP =,E15.5,
     1 2X4HXO =,F15.5,2X6HWLIM =,E15.5/1H0,9X,1H!,6X4HU(2),6X4HU(3),
     2 6X4HU(4),6X4HU(5),6X4HU(6),6X4HU(7),6X4HU(8),6X4HU(9),
     3 5X5HU(10),5X5HU(11),5X,5HOMEGA,10H  LIQ SPIN)
C
C*** COMPUTE CONSTANTS
      NM1=N-1
      X(1)=XO
      SUM=XO/2.
      AMBDA=1./DX**2
      DO 2 J=2,N
      X(J)=DX*FLOAT(J-1)+XO
      SUM=SUM+X(J)
    2 CONTINUE
      SUM=SUM X(N)/2.
      B=1./SUM
      C2=0.0288*TCON
      C1=-C2*0.2808
C
C*** INITIALIZE STATE VECTOR
      U(1)=WLIM
      DO 4 J=2,NM1
      U(J)=WLIM
    4 CONTINUE
      U(N)=1.0
C
C*** INITIALIZE RUNGE-KUTTA SUBROUTINE
C*** SOLVE DIFF EQNS FOR DERIVATIVE ICS
      T=0.0
      CALL DIFEQ(T,U,4)
   19 KOUNT=0
C
C*** START OF SOLUTION LOOP
   20 CONTINUE
C
C*** MOVE STATE FROM T TO T+DT
      CALL KUTTA(T,DT,U,WP,11,2,DIFEQ)
C
C*** COMPUTE EFFECTIVE ANGULAR VELOCITY
      SUM=0.0
      DO 30 J=1,NM1
      SUM=SUM+(U(J)+U(J+1))*(X(J)+X(J+1))/4.
   30 CONTINUE
      OMEGA=B*SUM
      FSPIN=923.*OMEGA
      TP=TCON*T+TO
      IF(TP.GT.66.) CALL EXIT
      KOUNT=KOUNT+1
```

```
      IF(KOUNT.EQ.NPRINT) GO TO 40
      IF(T.GT. 1.0) CALL EXIT
      GO TO 20
   40 WRITE (6,300)TP,(U(I),I=2,11),OMEGA,FSPIN
  300 FORMAT(1H ,13F10.5)
      IF(T.GT.1.0) CALL EXIT
      GO TO 19
      END

      SUBROUTINE DIFEQ(TIME,U,KUTTA)
C
C**** DIFFERENTIAL EQUATIONS FOR ANGULAR VEL. IN CONFIG. A
      DIMENSION U(22),X(11)
      COMMON AMBDA,N,X,DX,C1,C2,TO
      U(N+1)=AMBDA*(U(2)-U(1))*((X(2)/X(1))**2+1.)/2.
      NM1=N-1
      WO=1.
      DO 10 I=2,NM1
      WM1=(1.-DX/X(I))
      WP1=1.+DX/X(I)
      U(I+N)=AMBDA*(U(I-1)*WM1-2.*U(I)*WO+U(I+1)*WP1)
   10 CONTINUE
      U( *N)=0.0
      RETURN
      END
```

```
      SUBROUTINE KUTTA(T,DT,V,W,NEQ,NORDP1,DIFFEQ)
      DIMENSION V(NEQ,NORDP1),W(NEQ,NORDP1,4)
      DT2=DT*0.5
      DT6=DT/6.0
      DO 1 I=1,NEQ
      DO 1 J=1,NORDP1
    1 W(I,J,1)=V(I,J)
      DO 2 K=1,3
      L=K+1
      GO TO (3,3,4),K
    3 DTW=DT2
      GO TO 5
    4 DTW=DT
    5 TW=T+DTW
      DO 6 I=1,NEQ
      DO 6 J=2,NORDP1
      J1=J-1
      WP=W(I,J1,1)+W(I,J,K)*DTW
      V(I,J1)=WP
    6 W(I,J1,L)=WP
      CALL DIFFEQ(TW,V,K)
      DO 2 I=1,NEQ
    2 W(I,NORDP1,L)=V(I,NORDP1)
      DO 7 J=2,NORDP1
      J1=J-1
      DO 7 I=1,NEQ
    7 V(I,J1)=W(I,J1,1)+DT6*(W(I,J,1)+2.0*(W(I,J,2)+W(I,J,3))+W(I,J,4))
      T=TW
      CALL DIFFEQ(T,V,4)
      RETURN
      END
```

DYNAMICS OF SPINUP OF LIQUID CONFIGURATION B, CASE 1

TIME STEP = 0.10000E-03 SPACE STEP = 0.67000E-01 XO = 0.33000E 00 WLIM = 0.20000E-01

	ξ = .397				.665		.799		.993			
T	U(2)	U(3)	U(4)	U(5)	U(6)	U(7)	U(8)	U(9)	U(10)	U(11)	OMEGA	LIQ SPIN
0.00100	0.02000	0.02000	0.02000	0.02000	0.02000	0.02010	0.02164	0.04114	0.21021	1.00000	0.12086	111.55720
0.00200	0.02000	0.02000	0.02000	0.02001	0.02011	0.02112	0.02967	0.08531	0.33796	1.00000	0.14563	134.41400
0.00300	0.02000	0.02000	0.02001	0.02007	0.02058	0.02414	0.04462	0.13608	0.42788	1.00000	0.16704	154.17460
0.00400	0.02000	0.02000	0.02004	0.02027	0.02176	0.02973	0.06482	0.18634	0.49396	1.00000	0.18604	171.71030
0.00500	0.02000	0.02002	0.02013	0.02076	0.02397	0.03793	0.08839	0.23328	0.54439	1.00000	0.20323	187.58050
0.00600	0.02001	0.02006	0.02033	0.02166	0.02737	0.04849	0.11375	0.27602	0.58415	1.00000	0.21902	202.15900
0.00700	0.02003	0.02015	0.02071	0.02310	0.03204	0.06098	0.13980	0.31455	0.61637	1.00000	0.23370	215.70520
0.00800	0.02007	0.02031	0.02133	0.02517	0.03796	0.07496	0.16577	0.34919	0.64308	1.00000	0.24746	228.40540
0.00900	0.02014	0.02059	0.02226	0.02793	0.04503	0.09003	0.19118	0.38036	0.66565	1.00000	0.26045	240.39700
0.01000	0.02027	0.02101	0.02354	0.03140	0.05313	0.10583	0.21576	0.40850	0.68503	1.00000	0.27279	251.78510
0.01100	0.02046	0.02161	0.02523	0.03558	0.06212	0.12207	0.23935	0.43399	0.70189	1.00000	0.28456	262.65110
0.01200	0.02075	0.02242	0.02734	0.04044	0.07185	0.13853	0.26187	0.45719	0.71674	1.00000	0.29584	273.05980
0.01300	0.02116	0.02348	0.02989	0.04594	0.08219	0.15502	0.28332	0.47838	0.72993	1.00000	0.30668	283.06340
0.01400	0.02170	0.02481	0.03290	0.05203	0.09301	0.17141	0.30371	0.49783	0.74176	1.00000	0.31712	292.70480
0.01500	0.02240	0.02642	0.03634	0.05865	0.10419	0.18759	0.32307	0.51574	0.75244	1.00000	0.32722	302.01950
0.01600	0.02328	0.02834	0.04022	0.06575	0.11564	0.20351	0.34146	0.53230	0.76214	1.00000	0.33699	311.03780
0.01700	0.02436	0.03056	0.04452	0.07325	0.12727	0.21910	0.35893	0.54766	0.77101	1.00000	0.34646	319.78490
0.01800	0.02564	0.03309	0.04920	0.08112	0.13901	0.23433	0.37554	0.56197	0.77916	1.00000	0.35567	328.28240
0.01900	0.02716	0.03593	0.05425	0.08929	0.15080	0.24918	0.39134	0.57532	0.78668	1.00000	0.36463	336.54900
0.02000	0.02890	0.03908	0.05964	0.09772	0.16260	0.26364	0.40638	0.58782	0.79364	1.00000	0.37335	344.60180
0.02100	0.03089	0.04253	0.06534	0.10636	0.17435	0.27770	0.42071	0.59956	0.80012	1.00000	0.38186	352.45430
0.02200	0.03312	0.04627	0.07133	0.11517	0.18603	0.29136	0.43438	0.61060	0.80617	1.00000	0.39016	360.11910
0.02300	0.03561	0.05029	0.07757	0.12411	0.19761	0.30462	0.44744	0.62101	0.81182	1.00000	0.39828	367.60830
0.02400	0.03834	0.05458	0.08405	0.13316	0.20906	0.31750	0.45992	0.63086	0.81713	1.00000	0.40621	374.93110
0.02500	0.04133	0.05912	0.09073	0.14228	0.22037	0.33000	0.47186	0.64018	0.82213	1.00000	0.41397	382.09660
0.02600	0.04456	0.06391	0.09760	0.15145	0.23153	0.34212	0.48330	0.64902	0.82684	1.00000	0.42157	389.11270
0.02700	0.04803	0.06892	0.10463	0.16064	0.24252	0.35389	0.49427	0.65742	0.83129	1.00000	0.42902	395.98730
0.02800	0.05174	0.07414	0.11180	0.16985	0.25334	0.36532	0.50480	0.66542	0.83551	1.00000	0.43632	402.72600
0.02900	0.05568	0.07957	0.11909	0.17904	0.26398	0.37640	0.51491	0.67305	0.83951	1.00000	0.44348	409.33590
0.03000	0.05984	0.08518	0.12649	0.18821	0.27444	0.38717	0.52463	0.68033	0.84331	1.00000	0.45051	415.82170
0.03100	0.06422	0.09096	0.13397	0.19734	0.28472	0.39763	0.53399	0.68729	0.84693	1.00000	0.45741	422.18890
0.03200	0.06880	0.09691	0.14153	0.20643	0.29481	0.40779	0.54301	0.69395	0.85039	1.00000	0.46418	428.44180
0.03300	0.07358	0.10300	0.14915	0.21545	0.30471	0.41766	0.55170	0.70034	0.85369	1.00000	0.47084	434.58490
0.03400	0.07855	0.10922	0.15682	0.22442	0.31444	0.42726	0.56008	0.70646	0.85684	1.00000	0.47738	440.62250
0.03500	0.08370	0.11557	0.16453	0.23331	0.32398	0.43659	0.56817	0.71235	0.85986	1.00000	0.48381	446.55780
0.03600	0.08902	0.12203	0.17227	0.24213	0.33334	0.44567	0.57600	0.71801	0.86276	1.00000	0.49014	452.39470
0.03700	0.09450	0.12860	0.18002	0.25086	0.34252	0.45451	0.58356	0.72346	0.86554	1.00000	0.49636	458.13590
0.03800	0.10013	0.13525	0.18779	0.25951	0.35154	0.46311	0.59088	0.72871	0.86821	1.00000	0.50248	463.78510
0.03900	0.10590	0.14199	0.19556	0.26808	0.36038	0.47149	0.59797	0.73377	0.87078	1.00000	0.50850	469.34440
0.04000	0.11181	0.14880	0.20332	0.27655	0.36905	0.47965	0.60483	0.73865	0.87326	1.00000	0.51443	474.81710
0.04100	0.11783	0.15568	0.21107	0.28493	0.37756	0.48760	0.61149	0.74337	0.87564	1.00000	0.52027	480.20530
0.04200	0.12397	0.16261	0.21881	0.29322	0.38591	0.49536	0.61795	0.74794	0.87794	1.00000	0.52602	485.51190
0.04300	0.13022	0.16960	0.22653	0.30142	0.39410	0.50292	0.62422	0.75235	0.88017	1.00000	0.53168	490.73820
0.04400	0.13656	0.17663	0.23422	0.30951	0.40214	0.51030	0.63031	0.75662	0.88231	1.00000	0.53726	495.88740
0.04500	0.14300	0.18369	0.24188	0.31752	0.41004	0.51751	0.63623	0.76076	0.88439	1.00000	0.54275	500.96060
0.04600	0.14951	0.19078	0.24951	0.32543	0.41778	0.52454	0.64199	0.76478	0.88640	1.00000	0.54817	505.96060
0.04700	0.15610	0.19790	0.25711	0.33324	0.42539	0.53141	0.64759	0.76867	0.88834	1.00000	0.55351	510.88840
0.04800	0.16275	0.20503	0.26466	0.34096	0.43286	0.53813	0.65304	0.77245	0.89023	1.00000	0.55877	515.74600
0.04900	0.16947	0.21218	0.27218	0.34859	0.44020	0.54469	0.65835	0.77612	0.89206	1.00000	0.56396	520.53540
0.05000	0.17623	0.21934	0.27964	0.35612	0.44740	0.55111	0.66352	0.77969	0.89383	1.00000	0.56908	525.25750
0.05100	0.18305	0.22650	0.28707	0.36356	0.45448	0.55739	0.66856	0.78316	0.89555	1.00000	0.57412	529.91470
0.05200	0.18990	0.23366	0.29444	0.37091	0.46144	0.56353	0.67348	0.78653	0.89723	1.00000	0.57910	534.50750
0.05300	0.19679	0.24081	0.30177	0.37816	0.46828	0.56954	0.67828	0.78982	0.89885	1.00000	0.58401	539.03800
0.05400	0.20370	0.24795	0.30904	0.38533	0.47500	0.57542	0.68296	0.79302	0.90044	1.00000	0.58885	543.50700
0.05500	0.21064	0.25509	0.31626	0.39241	0.48160	0.58119	0.68753	0.79614	0.90198	1.00000	0.59363	547.91650

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Computer Program for Modified Point-Mass Exterior Ballistics

The program used to determine the ballistic differential effects of changes in inertial parameters is a general-purpose, exterior ballistics program capable of simulating the flight of gun-boosted rockets as well as purely ballistic systems.

The program employs two alternative ways of treating the aerodynamic forces normal to the body. The first option simply assumes trailing behavior, that is, that yaw is always zero. This option is inadequate for the purpose of this study. The second option computes an equilibrium yaw, or "yaw of repose," necessary to precess the velocity vector in the vertical plane at the proper rate. Using the yaw of repose, normal body forces are computed and resolved into components in an inertial frame of reference. These are added to the drag, gravitational, and Coriolis forces to complete a point-mass description. This procedure gives satisfactory agreement with experiment, providing accurate aerodynamic data exist and that the projectile displays adequate stability. This option is exercised by setting switch IOPTY equal to unity.

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SSI=0275044B

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C
C      EXTERIOR BALLISTICS OF BOOSTED ROCKETS
C      A THREE-OEGREE-OF-FREEOOM MOOEL APPLICABLE WHERE
C      TRAILING OR FOLLOWING BEHAVIOR CAN BE ASSUMED
C
C
       REAL RS(401),TS(401),VS(401),CJRT(11),CJVT(11)
       DIMENSION TITLE(20),U(12),WP(48),XMTBL(11),CDTBL(11),COTBL(11)
      1,TMACH(11),TKA(11),TKOYAW(11),TKL(11),TKM(11),TKF(11),
      2 TKT(11),TKH(11),TKS(11),TCP(11)
       INTEGER *2 CHAR(1)/'*'/
       DATA RE/6.378E6/,OMEGA/0.72915E-4/
C      ASSIGN CONSTANTS NEEOEO BY OIFFERENTIAL EQUATIONS TO COMMON
C
       COMMON EMO,EMB,SPI,FC,BRATE,OELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,
      1 VW,VCW,ALT,R,IEND,CMACH,REYNLD,RESIS,CAL,DLONG,IOPTY,YAW,AMOM,
      2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIOE
       COMMON /SRCOM/TM
       COMMON/COFCOM/XCG,SMARG,EM,THIO,PRNU ,ALTRIM,CNATRM,GLIOE,EPSTHE
      1 ,STAFAC,YAWNU,TKA,TKDYAW,TKL,TKM,TKF,TKT,TKH,TKS,TCP,TMACH,NARTBL
       COMMON/WINCOM/RWC,XWC
       COMMON/DRGCOM/      XMTBL,CDTBL,COTBL,NTBL
       COMMON/SNOCOM/CAOENS
C
C****  TABLES OF AEROOYNAMIC COEFFICIENTS IS PARAM. SET INPUT SET -1.
C****  IF PARAMETERS NTBL AND NARTBL ARE BOTH ZERO, ENOOGENOUS
C****  FUNCTIONAL FITS TO THE AEROOYNAMIC TABLES(WITH THE T387 FORM)
C****  WILL BE USED.  SEE SUBROUTINE ACOEFS.
C****  IF ONLY NARTBL IS ZERO, THE ZERO-LIFT ORAG TABLE IS REQUIREO
C****  WITHOUT REQUIRING TABLES FOR THE OTHER AERO COEFFICIENTS.
C****  IF CERTAIN AERO COEFFICIENTS ARE DEFINEO (KNOWN), THESE
C****  CAN BE READ WITH THE OTHERS LEFT BLANK.  THE PROGRAM WILL
C****  USE THE TABULATEO COEFFICIENTS ANO DEFAULT TO THE ENDOGENOUS
C****  FUNCTIONS FOR THOSE ENTERED AS ZERO.
C      O=PROJECTILE CALIBER, MILLIMETERS            PARAMETER INPUT 1
C      EMO=INITIAL PROJECTILE MASS, LBM                       INPUT 2
C      EMB=BURNT MASS, LBM                                    INPUT 3
C      FC=NOMINAL THRUST LEVEL, LBF                           INPUT 4
C      SPI=SPECIFIC IMPULSE OF ROCKET PROPELLANT, LBF/LBM/SEC INPUT 5
C      BRATE=PROPELLANT BURNING RATE, LBM/SEC      ENDOGENOUS VARIABLE
C      OELT=THRUST RISE TIME, SEC                             INPUT 6
C      TO=IGNITION TIME FOR ROCKET MOTOR           ENDOGENOUS VARIABLE
C      IN SUBROUTINE 'BURN' THE THRUST OECAY TIME IS ASSUMEO
C      EQUAL TO THE THRUST RISE TIME.  A TYPICAL VALUE = 0.1 SEC.
C      TB=EFFECTIVE BURNING INTERVAL, SEC          ENOOGENOUS VARIABLE
C      ISW= A SWITCH SIGNALING COMMENCEMENT OF BURNING   ENDO. VARIABLE
C      IENO=A SWITCH SIGNALING ENO OF BURNING            ENDO. VARIABLE
C      V=PROJECTILE VELOCITY, M/SEC                      ENDO. VARIABLE
C      VO=MUZZLE VELOCITY OF THE PROJECTILE, FT/SEC.          INPUT 7
C      THETA=ATTITUDE OF PROJECTILE, OEG                 ENDO. VARIABLE
```

```
C     CALSQ=CALIBER SQUAREO, M**2                          ENOO. VARIABLE00005100
C     VW=VELOCITY OF HEADWIND, M/SEC (READ IN IN FT/SEC)   ENDO. VARIABLE00005200
C     VWF=VELOCITY OF HEAOWINU IN FT/SEC                         INPUT 8 00005300
C     HO=INITIAL ALTITUDE, FT.                                   INPUT 9 00005400
C     HTERM=TERMINAL ALTITUOE, FT.                               INPUT 1000005500
C     FFCTR=FORM FACTOR RELATIVE TO PROGRAMMEO DRAG FUNCTION.    INPUT 1100005600
C     SEE FUNCTION 'ORAG' FOR THE SPECIFIC ORAG FUNCTION USEO.         00005700
C     QEO=INITIAL QUADRANT ELEVATION, DEG.                       INPUT 1200005800
C     OQE=QUADRANT ELEVATION INCREMENT, OEG.                     INPUT 1300005900
C     STEP=TIME STEP IN NUMERICAL INTEGRATION PROCEOURE, SEC.    INPUT 1400006000
C     TM=TIME AT WHICH BURNING OF ROCKET MOTOR SHOULO COMMENCE.  INPUT 1500006100
C     NQE=NUMBER OF INCREMENTS OF QUADRANT ELEVATION             INPUT 1600006200
C     NPRINT=NUMBER OF TIME STEPS EXECUTEO BETWEEN PRINTS        INPUT 1700006300
      CONTINUE                                                         00006400
C     ALT=TRUE ALTITUOE ABOVE SEA LEVEL, M                 ENOO. VARIABLE00006500
C     R=RAOIUS FROM CENTER OF EARTH TO PROJECTILE , M      ENDO. VARIABLE00006600
C     RE=NOMINAL RAOIUS OF THE EARTH AT THE EQUATOR, M           CONSTANT00006700
C     OMEGA=ANGULAR VELOCITY OF THE EARTH, RAD/SEC               CONSTANT00006800
C     IOPTY= A SWITCH INOICATING CHOICE OF YAW OPTION.           INPUT 1800006900
C     IOPTY= 1 PROOUCES COMPUTATION OF YAW OF REPOSE FOR SPINNING PROJEC00007000
C     IOPTY= 0 SIGNIFIES A TRAILING PROJECTILE WITHOUT SPIN.  FOR      00007100
C     THIS OPTION THE FOLLOWING INPUTS ARE UNNECESSARY.                00007200
C     SPINO=INITIAL SPIN, RAO/SEC                                INPUT 1900007300
C     XCG=POSITION OF CENTER OF GRAVITY AFT OF NOSE, CALIBERS    INPUT 2000007400
C     XCP=POSITION OF CENTER OF PRESSURE AFT OF NOSE, CAL ENOO. VARIABLE00007500
C     CLONG=PROJECTILE LENGHT IN CALIBERS.                       INPUT 2100007600
C     AMOM=LONGITUOINAL MOMENT OF INERTIA OF THE PROJECTILE, KG*M**2   00007700
C                                                                INPUT 2200007800
C     BMOM=TRANSVERSE MOMENT OF INERTIA OF THE PROJECTILE, KG*M**2     00007900
C                                                                INPUT 2300008000
C     VCW=VELOCITY OF CROSSWINO FROM RIGHT LOOKING OOWNRANGE(REAO IN FT/0000B100
C                                                                INPUT 2400008200
C     WTAREA=WETTEO AREA RATIO USEO IN COMPUTING SKIN                  0000B300
C     FRICTION ORAG                                              INPUT 2500008400
C     ISEP=A SWITCH INOICATING CHOICE OF SEPARATE                      00008500
C     COMPUTATION OF SKIN FRICTION DRAG.                         INPUT 2600008600
C     = 1 IF FRICTION ORAG IS COMPUTEO SEPARATELY ANO ADOEO TO FORM ORAG0000B700
C     = 0 IF FRICTION ORAG IS INCLUDEO IN DRAG FUNCTION.               00008800
C     SMARG=PROJECTILE STATIC MARGIN, CAL.                 ENOO. VARIABLE00008900
C     PSI=ANGULAR ORIENTATION OF YAW VECTOR                ENDO. VARIABLE00009000
C     SRNG=SLANT RANGE TO PROJECTILE POSITION, M           ENOO. VARIABLE00009100
C     VHXF=VELOCITY OF LAUNCHER IN RANGEWISE DIRECTION.  (FT/SEC)      00009200
C     VHYF=VELOCITY OF LAUNCHER IN VERTICAL OIRECTION.  (FT/SEC)       00009300
      CONTINUE                                                         00009400
C     RWC IS HEAOWINO COEF.                                            00009500
C     XWG IS CROSSWINO COEF.                                           00009600
C     PSI MAY BE COMPUTEO BY REMOVING 'C' S FROM COMMENT CAROS         00009700
C     IN SUBROUTINE FLIGHT.                                            00009800
C     CAOENS IS THE CORRECTION FACTOR FOR AIR OENSITY RELATIVE TO STANOA00009900
C                                                                      00010000
      EXTERNAL FLIGHT                                                  00010100
      EQUIVALENCE (U(1),X),(U(2),Y),(U(3),Z),(U(4),SPIN),              00010200
```

```
      1 (U(5),XO),(U(6),YO),(U(7),ZO),(U(8),SPINO),
      2 (U(9),XDD),(U(10),YDD),(U(11),ZDD),(U(12),SDD)
C       READ IN RUN DESCRIPTION, CONSTANTS IN FLIGHT EQUATIONS AND
C       INITIAL CONDITIONS.
C
        READ (5,256,END=30) TITLE,NTBL,NARTBL
   256  FORMAT(20A4/2I2)
        IF(NTBL.EQ.0) GO TO 255
        READ (5,250) (XMTBL(I),I=1,NTBL)
        READ (5,250) (CDTBL(I),I=1,NTBL)
        READ (5,250) (CDTBL(I),I=1,NTBL)
   250  FORMAT(8F10.0)
        WRITE (6,252) TITLE
   252  FORMAT(1H1,20A4/1H0,10H   MACH NO,10H COEF DRAG,10H DRAG INCR)
        DO 253 I=1,NTBL
        WRITE (6,254) XMTBL(I),CDTBL(I),CDTBL(I)
   253  CONTINUE
        IF(NARTBL.EQ.0)     GO TO 255
        READ (5,250) (TMACH(I),I=1,NARTBL)
        READ (5,250) (TKA(I),I=1,NARTBL)
        READ (5,250) (TKDYAW(I),I=1,NARTBL)
        READ (5,250) (TKL(I),I=1,NARTBL)
        READ (5,250) (TKM(I),I=1,NARTBL)
        READ (5,250) (TCP(I),I=1,NARTBL)
        READ (5,250) (TKF(I),I=1,NARTBL)
        READ (5,250) (TKT(I),I=1,NARTBL)
        READ (5,250) (TKH(I),I=1,NARTBL)
        READ (5,250) (TKS(I),I=1,NARTBL)
   254  FORMAT(1H ,3F10.4)
        WRITE (6,272)
   272  FORMAT(1H0,10H   MACH NO,8X,2HKA,5X,5HKDYAW,
      1 8X,2HKL,8X,2HKM,10H    CP, CAL)
        DO 257 I=1,NARTBL
        WRITE (6,270) TMACH(I),TKA(I),TKDYAW(I),TKL(I),TKM(I),TCP(I)
   270  FORMAT(1H ,6F10.5)
   257  CONTINUE
   255  CONTINUE
C****   PROVISIONAL CONSTANT GLIDE ANGLE JAN 75
C****   SWITCH IGLIDE MUST BE SET TO 1 FOR CONST. GLIDE ANGLE TRAJECTORY.
        READ (5,264) IGLIDE,ALTRIM,CNATRM ,GLIDE,EPSTHE
   264  FORMAT (I1,9X,4F10.0)
        IF(IGLIDE.NE.1) GO TO 1
        WRITE (6,268)
   268  FORMAT(1H0,15H  TRIM ANGLE, R,6X,9HCNA(TRIM),
      1 15H GLIDE ANGLE, R,15H    TOLERENCE, R)
        WRITE (6,266) ALTRIM,CNATRM,GLIDE,EPSTHE
   266  FORMAT (1H ,4F15.5)
C****   PROVISIONAL CONSTANT GLIDE ANGLE JAN 75
     1  READ (5,2,END=30) TITLE,D,EMO,EMB,FC,SP1,DELT,VO,VWF,HO,HTERM,
      1 FFCTR,QEO,DQE,STEP,TM,NQE,NPRINT,IOPTY
     2  FORMAT(20A4/8F10.0/7F10.0,3I3)
C****   SWITCH NABLE IS SET FROM 0 TO 1 AT TIME TENABL
```

```
  258 READ (5,260) CADENS,VCW,TENABL,THID
  260 FORMAT(4F10.0)
      WRITE (6,262) TENABL,THID,CADENS
  262 FORMAT(1H0,14HENABLE TIME = ,F10.4,5X,
     1 21HTHRUST DRAG FACTOR = ,F10.4,5X,18HAIR DENS FACTOR = ,F10.4)
      IF(IOPTY.NE.1) GO TO 25
      READ 26,SPIND,XCG,CLONG,AMOM,BMOM,WTAREA,ISEP
      READ 26,VHXF,VHYF,RWC,XWC
   26 FORMAT(6F10.0,I2)
      GO TO 27
   25 SPIND=0.0
      XCG=0.
      CLONG=0.
      AMOM=0.
      BMOM=0.
      WTAREA=0.
      ISEP=0
      VHXF=0.0
      VHYF=0.0
      RWC=0.0
      XWC=0.0
      SMARG=0.0
      STAFAC=0.0
      YAWNU=0.0
      PRNU=0.0
      PSI=0.0
      OMY=0.0
C     START QUAD-ELEV LOOP
   27 QE=QEO-DQE
      SDO=0.0
      CAL=0*1.E-3
      DO 3 IQE=1,NQE
      QE=QE+DQE
      THETA=QE/57.29578
      TD=1.E10
      EM=EMO
C     IEND IS A SWITCH SIGNALLING END OF BURNING
      IEND=0
C     ASSIGN TIME INCREMENT FOR INTEGRATION.
C
      DT=STEP
C     NABLE IS A SWITCH SIGNALING CONTROLS DEPLOYED FOR GUIDED FLIGHT.
      NABLE=0
C    PRINT AND LABEL RUN DESCRIPTION, CONSTANTS, AND
C     INITIAL CONDITIONS
      PRINT 9,TITLE,FFCTR,VD,EMO,EMB,D,QE,OT,FC,SPI,VWF,HD,HTERM,VCW
     1,RWC,XWC
    9 FORMAT(1H120A4/1H010X5HFFCTR13X2HVO13X2HMO13X2HMB14X1HD/
     1 1H 5F15.6/1H0,
     2 6X9HQUAD ELEV8X7HTM STEP9X6HTHRUST5X10HSP IMPULSE9X6HV-WIND/
     3 1H 5F15.6/1H04X11HINIT ALT,FT4X11HTERM ALT,FT15H VEL XWIND,FT/S,
     4 12X,3HRWC,12X,3HXWC,/1H 5F15.6)
```

```
      PRINT 91,VHXF,VHYF
   91 FORMAT(/8H VHXF = ,F12.2,20H FT/SEC       VHYF = ,F12.2,7H FT/SEC,/
     1/)
      PRINT 92,DELT,TM
   92 FORMAT(1H 19HTHRUST RISE TIME = ,F12.4,5H  SEC,8H   TM = ,F12.4,
     1 5H  SEC)
C
C
C     YO=INITIAL ALTITUDE, M
C     HO=ALTITUDE READ IN IN FT
C     YTERM=TERMINAL ALTITUDE, M
C     HTERM=TERMINAL ALTITUDE READ IN IN FT
      IF(IOPTY.NE.1) GO To 29
      PRINT 28,XCG,CLONG,AMOM,BMOM
   28 FORMAT(1H0,11HLOC OF CG =,E10.4,2X,3HCAL,14H PROJ LENGTH =,E10.4,
     1 2X,3HCAL,15H AXIAL M OF I =,E11.5,2X,7HKG M**2,
     2 15H TRANS M OF I =,E11.5,2X,7HKG M**2)
   29 IF(SPI.EQ.0.0) GO TO 60
      BRATE=FC/SPI
      IF(BRATE.EQ.0.0) GO TO 60
      TB=(EMO-EMB)/BRATE
      PRINT 40, TB
   40 FORMAT(1H0,22HEFFECTIVE BURN TIME = ,F10.4,4H SEC)
      GO TO 61
   60 TB=0.
      BRATE=0.
C
C     COMPUTE AUXILLIARY CONSTANTS AND REDIMENSION INPUTS
   61 CALSQ=D**2*1.E-6
      DLONG=D*1.0E-3*CLONG
      VELO=0.3048*VO
C     SUPVEL IS THE SUPREMIUM OF PROJECTILE VELOCITY.
      SUPVEL=VELO
C     SUPALT IS SUPREMIUM OF PROJECTILE ALTITUDE.
      SUPALT=0.0
      VW=0.3048*VWF
      VCW=0.3048*VCW
      YO=0.3048*HO
      YTERM=0.3048*HTERM
      RTERM=RE+YTERM
C     INITIALIZE TIME, X, X-DOT, Y AND Y-DOT.
C
      T=0.
      X=0.
      Y=YO
      Z=0.
      SPIN=SPINO
      VHX=.3048*VHXF
      VHY=.3048*VHYF
      XD=VELO*COS(THETA)+VHX
      YD=VELO*SIN(THETA)+VHY
      THETA=57.29578*ATAN(YD/XD)
```

```
      V=SQRT(XD**2+YD**2)
      ZD=0.
      SPIND=0.0
      XDD=0.0
      YDD=0.0
      ZDD=0.0
      YAW=0.0
      ALT=YO
      SRNG=0.0
      ISW=IBURN(T,ALT,THETA,V)
      IF(ISW.EQ.1) GO TO 70
      GO TO 71
   70 TO=0.0
      DT=STEP/4.
C
C     INITIALIZE RUNGE-KUTTA SUBROUTINE
   71 CALL RUNGE1(U,WP,4,2,FLIGHT)
C     SOLVE FLIGHT EQUATIONS FOR INITIAL CONDITIONS.
C
      CALL FLIGHT(T,U,4)
C
C     INITIALIZE COUNTER FOR DETERMINING NUMBER OF POINTS TO BE PLOTTED
C     AT END OF TRAJECTORY SOLUTION.
C
      NPLOT=0
C
C
C     INITIALIZE COUNTER FOR COUNTING LINES PER PAGE.
C
      LINE=0
C
      IPRINT=-NPRINT
      YAWDEG=YAW*57.3
C     GO TO PRINT OUT INITIAL CONDITIONS.
C
      GO TO 4
C
C     START OF SOLUTION LOOP.  SAVE LAST VALUES OF FLIGHT VARIABLES.
C
    5 XP=X
      RP=R
      ZP=Z
      XDP=XD
      YDP=YD
      VP=V
      THETAP=THETA
      CMACHP=CMACH
      RESISP=RESIS
      YAWP=YAWDEG
      SPINP=SPIN
      SMARGP=SMARG
      STAFAP=STAFAC
```

```
      PSIP=PSI
      SRNGP=SRNG
C
C     CALL RUNGE-KUTTA SUBROUTINE TO SOLVE FLIGHT EQUATIONS FROM
C     T TO T+DT.
C
      CALL RUNGE2(T,DT)
C
C
C
C     SAVE POSITIONAL COORDINATES OF PROJECTILE FOR LATER PLOTTING OF
C     TRAJECTORY.
C
C     SAVE RANGE AND FLIGHT TIME IN ARRAYS FOR SUBSEQUENT
C     POLYNOMIAL FIT
C
      SRNG=SQRT(X*X+(Y-YO)**2+Z*Z)
      IF(T.LT.TENABL) GO TO 520
      IF(NABLE.EQ.1) GO TO 520
      PRINT 530,T
  530 FORMAT(1H0,18HENABLEMENT TIME = ,F10.3)
      NABLE=1
C     ****************************************************************
C     IF (NPLOT.EQ.400) GO TO 520
C     IF (SRNG.GT.SRNGEM) GO TO 520
C     NPLOT=NPLOT+1
C     TS(NPLOT)=T
C     RS(NPLOT)=SRNG
C     VS(NPLOT)=V
C     ****************************************************************
  520 CONTINUE
C
      YAWDEG=YAW*57.3
      IF(V.GT.SUPVEL) SUPVEL=V
      IF(ALT.GT.SUPALT) SUPALT=ALT
   88 IF(ISW.EQ.0) GO TO 50
      IF(IENO.EQ.1) GO TO 51
      IF(T.GE.TO+TB+DELT) GO TO 49
      GO TO 51
   49 DT=STEP
      IENO=1
      CALL BURN(T,XMASS,THRUST)
      PRINT 80, XMASS,THRUST,ALT,V,T
   80 FORMAT(1H0,9H  MASS = ,F10.4,11H  THRUST = ,F10.4,
     1 13H  ALTITUDE = ,F10.2,10H  SPEED = ,F10.2,10H   TIME = ,F10.3)
      GO TO 51
   50 ISW=IBURN(T,ALT,THETA,V)
      IF(EMD.EQ.EMB) ISW=0
      IF(ISW.EQ.0) GO TO 51
      TO=T
      PRINT 20, TO
   20 FORMAT(1H0,15HBURN STARTS AT ,F10.4,4H SEC)
```

```
      DT=STEP/4.
   51 IPRINT=IPRINT+1
      IF(IPRINT.EQ.0) GO TO 53
      GO TO 52
   53 IPRINT=-NPRINT
C
C     ADVANCE LINES COUNTER AND CHECK IF TIME TO EJECT PAGE AND LABEL.
C
    4 LINE=LINE+1
      DMY=YAWNU
C**** DMY IS A DUMMY VARIABLE USED FOR OUTPUT OF CHOICE
      IF (LINE.LE.0) GO TO 6
      LINE=-50
      PRINT 7,TITLE
    7 FORMAT(1H120A4/1H0,9HTIME,SECS,7X3HX,M5X5HALT,M,7X,3HZ,M2X,
     1 8HXDDT,M/S2X8HYDDT,M/S5X5HV,M/S1X8HMACH NO.2X7HDRAG,LB,
     2 9H  THETA,D,9H    YAW,D,9H SPIN,R/S,9H STA FAC ,8H DUMMY V)
    6 PRINT 8,T,X,ALT,Z,XD,YD,V,CMACH,RESIS,THETA,YAWDEG,SPIN,STAFAC,DMY
    8 FORMAT(1H 1F9.3,3F10.1,3F10.1,F9.2,2F9.1,4F9.2)
      IF(T.GT.300.) GO TO 30
C     RULE FOR STOPPING SOLUTION - STOP WHEN PROJECTILE HITS GROUND.
C
   52 IF(.NOT.(R.LE.RTERM.AND.THETA.LT.0.0)) GO TO 5
C     INTERPOLATE SOLUTION VARIABLES FOR R=RTERM
C
      YE=YTERM
      TE=T-DT*(R-RTERM)/(R-RP)
      DEL=(T-TE)/DT
      XE=X-DEL*(X-XP)
      ZE=Z-DEL*(Z-ZP)
      XDE=XD-DEL*(XD-XDP)
      YDE=YD-DEL*(YD-YDP)
      VE=V-DEL*(V-VP)
      THETAE=THETA-DEL*(THETA-THETAP)
      CMACHE=CMACH-DEL*(CMACH-CMACHP)
      RESISE=RESIS-DEL*(RESIS-RESISP)
      YAWE=YAWDEG-DEL*(YAWDEG-YAWP)
      SPINE=SPIN-DEL*(SPIN-SPINP)
      STAFAE=STAFAC-DEL*(STAFAC-STAFAP)
      SMARGE=SMARG-DEL*(SMARG-SMARGP)
      PSIE=PSI-DEL*(PSI-PSIP)
      SRNGE=SRNG-DEL*(SRNG-SRNGP)
C     XPLOT(NPLOT)=XE
C     YPLOT(1,NPLOT)=YE
C
C     PRINT OUT SOLUTION VARIABLES FOR Y=YTERM.
C
      PRINT 8,TE,XE,YE,ZE,XDE,YDE,VE,CMACHE,RESISE,THETAE,YAWE,
     1 SPINE,STAFAE,SRNGE
C
      SUPVEL=SUPVEL/0.3048
      RANGE=SQRT(XE**2+ZE**2)
```

```
      RANGE=RANGE*(1.+(RANGE/RE)**2/6.)/1852.
C     PRINT MAXIMUM VELOCITY, RANGE, AND ALTITUDE.
      PRINT 90, SUPVEL, RANGE,SUPALT
   90 FORMAT(1H0,20HMAX PROJ VELOCITY = ,F15.4,4H F/S,
     1 3X,12HMAX RANGE = ,F15.4,11H NAUT MILES,3X,10HMAX ALT = ,
     2 F15.4,8H  METERS)
C
C     PLOT THE TRAJECTORY JUST COMPUTED.
C
C     LABEL PLOT WITH TITLE, QE AND VO.
C
C
      PRINT 10,TITLE,QE,VO
   10 FORMAT (1H015X20A4/4H QE=F5.1,10H DEG,  VO=F6.1,4H F/S)
    3 CONTINUE
C     ****************************************************************
C     CALL POLFIT(RS,TS,NPLOT,3,0,CJRT,G,G,.TRUE.,SDEV,
C    1 20HFLIGHT TIME VS RANGE)
C     CALL POLFIT(RS,VS,NPLOT,3,0,CJVT,H,HINV,.TRUE.,SOEV,
C    . 20HPROJ. VEL. VS. RANGE)
C     RETURN FOR ANOTHER CASE.
C     ****************************************************************
      GO TO 1
   30 CALL EXIT
      END
      SUBROUTINE SOUND(A,G,RHO,VISCO)
      COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,
     1 VW,VCW,ALT,R,IEND,CMACH,REYNLD,RESIS,CAL,OLONG,IOPTY,YAW,AMOM,
     2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIDE
      COMMON/SNDCOM/CADENS
      EQUIVALENCE (Y,ALT)
C
C     SUBROUTINE COMPUTES THE SPEED OF SOUND IN M/SEC
C     VERSUS ALTITUDE IN METERS.  ALSO COMPUTED IS THE
C     ACCELERATION DUE TO GRAVITY IN M/SEC/SEC AND THE
C     AIR DENSITY IN KG/M**3 AND THE ABSOLUTE VISCOSITY
C     OF THE AIR IN KG/M/SEC.  NOTE THAT REYNOLD'S NUMBER
C     PER METER IS GIVEN BY A*RHO*EMACH/VISCO.
C
      G=9.826*(6.378E6/(6.378E6+Y))**2
      O=6.356766E6+Y
      IF(Y.LE.11019.07) GO TO 1
      IF(Y.LE.20063.12) GO TO 2
      IF(Y.LE.32161.9) GO TO 3
      IF(Y.LE.47350.09) GO TO 4
      IF(Y.LE.52428.88) GO TO 5
      IF(Y.LE.61591.03) GO TO 6
      IF(Y.LE.79994.14) GO TO 7
      RHO=0.4636*EXP(-0.12207E-3*Y)
C     T=TEMPERATURE IN DEGREES KELVIN
      T=180.65
    8 A=20.053*SQRT(T)
```

```
      RHO=RHD*CAOENS
      VISCO=0.00467*(T+110.)*(T/217.78)**1.5
C     THIS IS THE SUTHERLAND VISCDSITY LAW.
      RETURN
    1 RHO=1.224999+Y*(-.1176033E-3+Y*(.433719E-8+Y*(-.7461659E-13
     1 +Y*(.5537603E-18-.9572727E-24*Y))))
      T=(1.831702E9-4.103083E4*Y)/O
      GO TO 8
    2 RHO=1.990142+Y*(-.2940114E-3+Y*(.1993974E-7+Y*(-.7637263E-12
     1 +Y*(.1615921E-16-.1476764E-21*Y))))
      T=216.65
      GO TO 8
    3 RHO=1.81561+Y*(-.235749E-3+Y*(.130807E-7+Y*(-.3819651E-12
     1 +Y*(.5798729E-17-.3626654E-22*Y))))
      T=(1.250058E9+6.553416E3*Y)/D
      GD TO 8
    4 RHO=1.10944+Y*(-.1140029E-3+Y*(.4817401E-8+Y*(-.1039241E-12
     1 +Y*(.1138793E-17-.5052135E-23*Y))))
      T=(8.839083E8+1.7938E4*Y)/O
      GD TO 8
    5 RHO=.8974979E-1+Y*(-.417905E-5+Y*(.3529753E-10+Y*(.1177144E-14
     1 +Y*(-.2567072E-19+.1449113E-24*Y))))
      T=270.65
      GO TO 8
    6 RHO=.1029082E-1+Y*(.1081853E-5+Y*(-.8523619E-10+Y*(.2075003E-14
     1 +Y*(-.2184824E-19+.860425E-25*Y))))
      T=(2.381562E9-1.233888E4*Y)/D
      GO TD 8
    7 RHO=0.4636*EXP(-0.12207E-3*Y)
      T=(3.157088E9-2.493041E4*Y)/O
      GO TO 8
      END
      SUBROUTINE FLIGHT(TIME,U,KUTTA)
      DIMENSION U(12)
      DIMENSION TMACH(11),TKA(11),TKDYAW(11),TKL(11),TKM(11),
     1 TKF(11),TKT(11),TKH(11),TKS(11),TCP(11)
      OATA RE/6.378E6/,OMEGA/0.72915E-4/,PIOFOR/.7853981/,TWOG/19.58418/
C     RE=NOMINAL RADIUS OF THE EARTH AT THE EQUATOR IN METERS
C     OMEGA=ANGULAR VELOCITY OF THE EARTH IN RADIANS/SEC
C
C     TABLE OF EQUIVALENCES
C     U(1) = X
C     U(2) = Y
C     U(3) = Z
C     U(4) = SPIN
C     U(5) = XDOT
C     U(6) = YOOT
C     U(7) = ZOOT
C     U(8) = SPIND
C     U(9) = XDBL
C     U(10) = YOBL
C     U(11) = ZDBL
```

```
C      U(12) = DUMMY
       EXTERNAL CDRAG
       COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,
      1VWO,VXW,ALT,R,IEND,CMACH,REYNLD,RESIS,CAL,DLONG,IOPTY,YAW,AMOM,
      2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIDE
       COMMON/COFCOM/XCG,SMARG,EM,THID,PRNU ,ALTRIM,CNATRM,GLIDE,EPSTHE
      1 ,STAFAC,YAWNU,TKA,TKOYAW,TKL,TKM,TKF,TKT,TKH,TKS,TCP,TMACH,NARTBL
       COMMON/WINCOM/RWC,XWC
       IF(ISW.EQ.0) GO TO 10
       IF(TIME.GT.TO+TB+DELT) GO TO 9
       CALL BURN(TIME,XMASS,THRUST)
       EM=XMASS
C      THRUST-INDUCED DRAG
       FFC=FFCTR*THID
       H=4.44823*THRUST
       TERMX=H*U(5)
       TERMY=H*U(6)
    12 VSQ=U(5)**2+U(6)**2+U(7)**2
       V=SQRT(VSQ)
       UP=RE+U(2)
       XSQ=U(1)**2
       YSQ=UP**2
       ZSQ=U(3)**2
       R=SQRT(XSQ+YSQ+ZSQ)
C
C      ALT=R-RE
       ALT=U(2)+XSQ/2./RE
       DRCOSX=U(1)/R
       DRCOSY=UP/R
       DRCOSZ=U(3)/R
       CALL SOUND(A,G,RHO,VISCO)
C
C      THIS GENERATES SPEED OF SOUND,GRAVITY,AIR DENSITY, AND VISCOSITY.
       IF(V.EQ.0.0) GO TO 13
       TERMX=TERMX/V
       TERMY=TERMY/V
C*****COMPUTE VELOCITY OF WIND AS A FUNCTION OF ALTITUDE.
C      HARG=1.000*U(2)
C      VW=VWO+RWC*VWC(HARG)
C      VCW=VXW+XWC*VWC(HARG)
       VW=VWO
       VCW=VXW
       VRELSQ=              ((U(5)+VW)**2+U(6)**2+(U(7)+VCW)**2)
       VREL=SQRT(VRELSQ)
       CMACH=VREL/A
C      COMPUTE REYNOLD'S NUMBER AND SKIN FRICTION COEFFICIENT (SFC).
       FRICT=0.0
       IF(DLONG.EQ.0.0) GO TO 48
       REYNLD=DLONG*A*RHO*CMACH/VISCO
       ALR=ALOG10(REYNLD)
       PWR=0.05*ALR
       SFC=0.455/ALR**2.58/(1.+0.2*CMACH**2)**PWR
```

```
      IF(ISEP.EQ.0) GO TO 48
      FRICT=WTAREA*SFC
C     DENOM=MASS OF PROJECTILE IN KG.
   48 DENOM=0.4536*EM
      DYNPRS=0.5*RHO*VRELSQ
      IF(IOPTY.EQ.1) GO TO 20
      YAW=0.0
      YAWSQ=0.0
      XKDYAW=0.0
      XTERMS=0.0
      YTERMS=0.0
      ZTERMS=0.0
      U(8)=0.0
   21 CALL CDRAG(CMACH,DRAG,CAD)
      COFDRG=FFC*DRAG+XKDYAW*YAWSQ+FRICT+CAO
C     THIS GENERATES THE CORRECTED COEFFICIENT OF DRAG
C
C     DIFFERENTIAL EQUATIONS
   22 CONTINUE
      FORM=PIOFDR*CALSQ*DYNPRS
      DRG=-COFDRG*FORM
C
C     RESIS IS AIR RESISTANCE IN POUNDS.
      RESIS=DRG             /4.44823
C**** PROVISIONAL CONSTANT GLIDE ANGLE JAN 75
      IF(IGLIDE.NE.1) GO TO 60
      THE=ATAN(U(6)/U(5))
      IF(NABLE.EQ.1) GO TO 62
      IF(THE.LE.GLIDE) NABLE=1
      HERR=ABS(THE-GLIDE)
      IF(HERR.LE.EPSTHE) NABLE=1
      IF(NABLE.NE.1) GO TO 60
   62 SAVE=0.5*ARSIN(DENOM*TWOG*COS(GLIDE)/FORM/CNATRM)
      ALPH=AMIN1(ALTRIM,SAVE)
      SAVE=CNATRM*FORM
      U(4)=SAVE*SIN(ALPH)/DENOM
C**** U(4) IS COMPUTED AS NORMAL ACCELERATION (M/S**2) INSTEAD OF SPIN.
      YAW=ALPH
      FL=0.5*SAVE*SIN(2.*ALPH)
      FDI=-SAVE*(SIN(ALPH))**2
      SINTH=U(6)/V
      COSTH=U(5)/V
      TERMX=TERMX-FL*SINTH+FDI*COSTH
      TERMY=TERMY+FL*COSTH+FDI*SINTH
   60 CONTINUE
C**** PROVISIONAL CONSTANT GLIDE ANGLE JAN 75
      U(10)=(DRG*U(6)/VREL+TERMY)/DENOM-G*DRCOSY+0.53166E-8*UP
     1 +2.*OMEGA*U(7)+YTERMS
      U(9)=(DRG*(U(5)+VW)/VREL +TERMX)/DENOM-G*DRCOSX+XTERMS
      U(11)=-2.*OMEGA*U(6)-G*DRCOSZ+ZTERMS+DRG*(U(7)+VCW)/VREL/DENOM
      U(12)=0.0
C     AC IS THE ACCELERATION OF THE PROJECTILE ALONG THE TRAJECTORY.
```

```
C        AC=(U(5)*U(9)+U(6)*U(10)+U(7)*U(11))/V
         IF(IOPTY.EQ.0) GO TO 14
         U(8)=-RHO*CALSQ**2/AMOM*XKA*U(4)*V
      14 IF(KUTTA.EQ.4) THETA=ARSIN(U(6)/V)*57.3
         RETURN
      13 U(9)=0.
         U(10)=-G
         THETA=90.
         RETURN
       9 EM=EMB
         GO TO 11
      10 EM=EMO
      11 TERMX=0.
         TERMY=0.
         FFC=FFCTR
         GO TO 12
      20 CALL ACOEFS(CMACH,YAW,XKA,XKDYAW,XKL,XKM,XKF,XKT,
        1XKH,XKS,XCP)
         VXRL=U(5)+VW
         VZRL=U(7)+VCW
         VRLSQ=VXRL**2+U(6)**2+VZRL**2
         VRL=SQRT(VRLSQ)
C        TEST FOR DYNAMIC STABILITY.
         BOTTOM=8.0*BMOM*DYNPRS*CAL**3*XKM
         STAFAC=(U(4)*AMOM)**2/BOTTOM
         IF(STAFAC.LE.1.0) GO TO 25
C****    COMPUTE THE YAWING FREQUENCY
         YAWNU=AMOM/BMOM*SQRT(1.-1./STAFAC)*U(4)/6.2832
C****    COMPUTE THE OVERTURNING MOMENT AND PRECESSIONAL FREQUENCY
         OTNMOM=2.*XKM*CAL**3*DYNPRS
         PRNU=OTNMOM/AMOM/U(4)/6.2832
C        COMPUTE YAW OF REPOSE.
         ALPHAB=RHO*CAL*VRLSQ*(XKL*XKM*VRLSQ+CALSQ*XKF*XKT*U(4)**2)
         IF(ABS(ALPHAB).LT.1.E-20) GO TO 25
         ALPHAA=AMOM*XKL*U(4)/CALSQ/ALPHAB
         ALPHAB=DENOM*XKT*U(4)/ALPHAB
         AMB=ALPHAB-ALPHAA
         ALPHAX=AMB*(U(6)*U(11)-VZRL*U(10))-ALPHAB*VZRL*G
         ALPHAY=AMB*(VZRL*U(9)-VXRL*U(11))
         ALPHAZ=AMB*(VXRL*U(10)-U(6)*U(9))+ALPHAB*VXRL*G
         YAWSQ=ALPHAX**2+ALPHAY**2+ALPHAZ**2
         YAW=SQRT(YAWSQ)
         IF(YAW.GT.1.5708) GO TO 25
         ARG=(VXRL*ALPHAZ-VZRL*ALPHAX)*VRL
         ARG1=(VXRL*ALPHAY-U(6)*ALPHAX)*VXRL-(U(6)*ALPHAZ-VZRL*ALPHAY)*VZRL
         IF(ABS(ARG1).LE.1.0E-20) GO TO 50
         PSI=57.3*ATAN(ARG/ARG1)
         GO TO 53
      50 IF(ARG*ARG1) 51,52,52
      51 PSI=-90.
         GO TO 53
      52 PSI=90.
```

```
   53 CONTINUE
C
C     PSI=ORIENTATION OF YAW.  THIS IS THE ANGLE BETWEEN THE PLANE
C     CONTAINING 8OTH THE VELOCITY ANO YAW VECTORS AND A VERTICAL
C     PLANE CONTAINING THE VELOCITY VECTOR.  IT IS MEASUREO
C     CLOCKWISE FROM THE VERTICAL PLANE.
C
C     END OF COMPUTATION OF YAW.
      OKFN=CAL*XKF*U(4)
      XKLVSQ=XKL*VRLSQ
      RDSQ=RHO*CALSQ/OENOM
      XTERMS=RDSQ*(XKLVSQ*ALPHAX+DKFN*(ALPHAY*VZRL-ALPHAZ*U(6)))
      YTERMS=ROSQ*(XKLVSQ*ALPHAY+DKFN*(ALPHAZ*VXRL-ALPHAX*VZRL))
      ZTERMS=RDSQ*(XKLVSQ*ALPHAZ+DKFN*(ALPHAX*U(6)-ALPHAY*VXRL))
      XKDYAW=2.54647*XKDYAW
C     ****************************
      IF(YAW.LT.0.69) GO TO 21
      PRINT 55,RHO,XTERMS,YTERMS,ZTERMS
   55 FORMAT(1H ,1P4E10.5)
C     ****************************
      GO TO 21
   25 PRINT 26,STAFAC
   26 FORMAT(1H0,40HUNSTA8LE PROJECTILE  STA8ILITY FACTOR = ,F10.4)
      CALL EXIT
      END
      SU8ROUTINE CDRAG(EMACH,DRAG,CAD)
C
C     PROGRAM COMPUTES THE COEFFICIENT OF ORAG VERSUS MACH NUMBER
C     AND THE THE COEFFICIENT INCREMENT DUE TO CANAROS.
C
      OIMENSION XMTBL(11),COT8L(11),COT8L(11)
      COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,
     1 VW,VCW,ALT,R,IENO,CMACH,REYNLO,RESIS,CAL,DLONG,IOPTY,YAW,AMOM,
     2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIDE
      COMMON/ORGCOM/       XMT8L,CDT8L,COT8L,NT8L
      IF(NTBL.NE.0) GO TO 5
      IF(EMACH.LE.0.80) GO TO 1
      IF(EMACH.LE.1.10) GO TO 3
      IF(EMACH.LE.3.0) GO TO 4
      EM3=EMACH-3.0
      DRAG=0.09+EM3*(-0.02+0.002*EM3)
      RETURN
    1 ORAG=0.0589
      RETURN
    3 C=10.*(EMACH-0.8)
      ORAG=0.07736*C**3*EXP(-C)+0.0589
      RETURN
    4 DRAG=0.21547+EMACH*(-0.05134+0.00317*EMACH)
    5 00 6 J=1,NTBL
      IF(EMACH.LT.XMTBL(J)) GO TO 8
    6 CONTINUE
    8 JL=J-1
```

```
      FRAC=(EMACH-XMTBL(JL))/(XMTBL(J)-XMTBL(JL))
      CD=CDTBL(JL)+(CDTBL(J)-CDTBL(JL))*FRAC
      CAO=0.0
      IF(NABLE.NE.1) GO TO 10
      CAO=COTBL(JL)+(COTBL(J)-COTBL(JL))*FRAC
   10 CONTINUE
      DRAG=CD
      CAD=CAO
      RETURN
      END
      SUBROUTINE ACOEFS(EMACH,YAW,XKA,XKDYAW,XKL,XKM,XKF,XKT,XKH,XKS,XCP
     1)
      DIMENSION TMACH(11),TKA(11),TKDYAW(11),TKL(11),TKM(11),TKF(11),
     1 TKT(11),TKH(11),TKS(11),TCP(11)
      COMMON/COFCOM/XCG,SMARG,EM,THIO,PRNU ,ALTRIM,CNATRM,GLIDE,EPSTHE
     1 ,STAFAC,YAWNU,TKA,TKDYAW,TKL,TKM,TKF,TKT,TKH,TKS,TCP,TMACH,NARTBL
C     EMACH = MACH NUMBER
C     XKA = SPIN DAMPING MOMENT COEFFICIENT
C     XKDYAW = YAW DRAG COEFFICIENT
C     XKL = LIFT FORCE COEFFICIENT
C     XKM = OVERTURNING MOMENT COEFFICIENT
C     XKF = MAGNUS FORCE COEFFICIENT
C     XKT = MAGNUS MOMENT COEFFICIENT
C     XKH = DAMPING MOMENT COEFFICIENT
C     XKS = PITCHING FORCE COEFFICIENT
C     XCP = CENTER OF PRESSURE AFT OF NOSE IN CALIBERS
C
C     FOR DEPENDENCE OF ACOEFS UPON YAW SEE BRL MEMO. RPT. NO. 2023
C     RELATIVE TO T387 TYPE PROJECTILE.
C     XKT=-0.14+0.0576*(EMACH-1.25)**2
      XKT=0.0
      XKF=0.157
      SYAW=SIN(YAW)**2
      GO TO 50
   51 CONTINUE
      DO 60 J=1,NARTBL
      IF(EMACH.LT.TMACH(J)) GO TO 70
   60 CONTINUE
   70 JL=J-1
      FRAC=(EMACH-TMACH(JL))/(TMACH(J)-TMACH(JL))
      IF(TKA(J).EQ.0.0) GO TO 52
      XKA=TKA(JL)+(TKA(J)-TKA(JL))*FRAC
   52 IF(TKDYAW(J).EQ.0.0) GO TO 53
      XKDYAW=TKDYAW(JL)+(TKDYAW(J)-TKDYAW(JL))*FRAC
   53 IF(TKL(J).EQ.0.0) GO TO 54
      XKL=TKL(JL)+(TKL(J)-TKL(JL))*FRAC
   54 IF(TKM(J).EQ.0.0) GO TO 55
      XKM=TKM(JL)+(TKM(J)-TKM(JL))*FRAC
   55 IF(TKF(J).EQ.0.0) GO TO 56
      XKF=TKF(JL)+(TKF(J)-TKF(JL))*FRAC
   56 IF(TKT(J).EQ.0.0) GO TO 57
      XKT=TKT(JL)+(TKT(J)-TKT(JL))*FRAC
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```
   57 IF(TKH(J).EQ.0.0) GO TO 58
      XKH=TKH(JL)+(TKH(J)-TKH(JL))*FRAC
   58 IF(TKS(J).EQ.0.0) GO TO 59
      XKS=TKS(JL)+(TKS(J)-TKS(JL))*FRAC
   59 IF(TKM(J).EQ.0.0) GO TO 62
      IF(XKL.EQ.0.0) CALL EXIT
      SMARG=-XKM/XKL
      XCP=XCG+SMARG
      RETURN
   62 IF(TCP(J).EQ.0.0) GO TO 63
      XCP=TCP(JL)+(TCP(J)-TCP(JL))*FRAC
   63 SMARG=XCP-XCG
      XKM=-XKL*SMARG
      RETURN
   50 CONTINUE
      IF(EMACH.LE.0.8) GO TO 10
      IF(EMACH.LE.0.9) GO TO 20
      IF(EMACH.LE.1.0) GO TO 30
      IF(EMACH.LE.1.1) GO TO 35
      IF(EMACH.LE.1.30) GO TO 40
      IF(EMACH.GT.1.5) GO TO 45
C     VALID FOR EMACH GTR 0.8 AND LT 1.5
    5 XKA=0.0038+0.002*EXP(-1.5*(EMACH-0.8))
C     VALID FOR EMACH GTR 0.9
    6 EM9=EMACH-0.9
      HOLD=1.-EXP(-5.*EM9)
      XKL=0.5507+0.4*HOLO
      XKL=XKL+6.6*SYAW
      XCP=0.237+1.57*HOLD
C     VALID FOR EMACH GTR 1.0
    7 XKS=-4.0+1.78*(EMACH-1.)
C     VALID FOR EMACH GTR 1.1
      XKDYAW=1.5+2.38*EXP(-2.72*(EMACH-1.1))
C     VALID FOR EMACH GTR 1.3
      XKH=3.7
C     VALID FOR ALL EMACH
    9 SMARG=XCP-XCG
      XKM=-XKL*SMARG
      IF(NARTBL.NE.0) GO TO 51
      RETURN
   10 XKA=0.0058
      XKDYAW=1.5
   11 XKL=0.62-0.077*EMACH
      XKL=XKL+4.3*SYAW
      XCP=1.2-1.07*EMACH
   12 XKS=-4.0
   13 XKH=0.71+2.3*EMACH
      GO TO 9
   20 EM8=EMACH-0.8
      XKA=0.0038+0.002*EXP(-1.5*EM8)
      XKDYAW=1.5+2.5*SIN(6.283*EM8)
      GO TO 11
```

```
   30 EM8=EMACH-0.8
      XKA=0.0038+0.002*EXP(-1.5*EM8)
      XKDYAW=1.5+2.5*SIN(6.283*EM8)
      EM9=EMACH-0.9
      HOLD=1.-EXP(-5.*EM9)
      XKL=0.5507+0.4*HOLD
      XKL=XKL+5.5*SYAW
      XCP=0.237+1.57*HOLD
      GO TO 12
   35 EM8=EMACH-0.8
      XKA=0.0038+0.002*EXP(-1.5*EM8)
      XKDYAW=1.5+2.5*SIN(6.283*EM8)
      EM9=EMACH-0.9
      HOLD=1.-EXP(-5.*EM9)
      XKL=0.5507+0.4*HOLD
      XKL=XKL+5.5*SYAW
      XCP=0.237+1.57*HOLD
      XKS=-5.78+1.78*EMACH
      GO TO 13
   40 EM8=EMACH-0.8
      XKA=0.0038+0.002*EXP(-1.5*EM8)
      XKDYAW=1.5+2.38*EXP(-2.72*(EMACH-1.1))
      EM9=EMACH-0.9
      HOLD=1.-EXP(-5.*EM9)
      XKL=0.5507+0.4*HOLD
      XKL=XKL+6.6*SYAW
      XCP=0.237+1.57*HOLD
      XKS=-5.78+1.78*EMACH
      GO TO 13
   45 XKA=0.0038+0.002*EXP(-1.5*(EMACH-0.8))
      GO TO 6
      END
      SUBROUTINE RUNGE1(V,W,NEQ,NORD,DIFEQ)
      DIMENSION V(12),W(48)
      NV=NEQ*NORD
      N=NV+NEQ
      RETURN
C
      ENTRY RUNGE2(T,DT)
      DT2=DT*.5
      DT6=DT/6.
      DO 1 I=1,N
    1 W(I)=V(I)
      DO 2 J=1,3
      NJM=NEQ+N*(J-1)
      JDECK=N*J
      IF (J-3)3,4,4
    3 DTW=DT2
      GO TO 5
    4 DTW=DT
    5 TW=T+DTW
      DO 6 I=1,NV
```

```
      K=I+JDECK
      L=I+NJM
      W(K)=W(I)+W(L)*DTW
    6 V(I)=W(K)
      CALL DIFEQ(TW,V,J)
      DO 2 I=1,N
      K=I+JOECK
    2 W(K)=V(I)
      00 7 I=1,NV
      K1=I+NEQ
      K2=K1+N
      K3=K2+N
      K4=K3+N
    7 V(I)=W(I)+DT6*(W(K1)+2.*(W(K2)+W(K3))+W(K4))
      T=TW
      CALL DIFEQ(T,V,4)
      RETURN
      END
      FUNCTION VWC(Y)
      IF(Y.GE.6700.)GOTO3
      VWC=5.1816+4.972E-5*Y+1.3494E-7*Y*Y
      RETURN
 3    VWC=10.058+3.9624*COS(.42946E-3*(Y-9448.8))
      IF(Y.GT.13700.) WRITE(6,1)
      RETURN
 1    FORMAT(28H ALTITUDE ABOVE 13700 METERS)
      ENO
      SUBROUTINE BURN(TIME,XMASS,THRUST)
C     SUBROUTINE COMPUTES PROJECTILE MASS IN POUNOS MASS ANO
C     ROCKET THRUST IN POUNDS FORCE
C     TO=TIME AT WHICH BURNING COMMENCES
C     EMO=INITIAL MASS, LBM
C     EMB=BURNT MASS, LBM
C     TIME=TIME AFTER LAUNCH ,SEC
C     SPI=SPECIFIC IMPULSE, LBF/LBM/SEC
C     FC=CONSTANT NOMINAL THRUST LEVEL, LBF
C     IBURN= INDICATOR OF COMMENCEMENT OF BURNING(IBURN=1)
C     DELT=RISE TIME OF THRUST--ASSUMED EQUAL TO DECAY TIME , SEC
C     BRATE=FC/SPI=BURNING RATE, LBM/SEC
C     TB=(EMO-EMB)/BRATE=EFFECTIVE BURNING TIME, SEC
C
      COMMON EMO,EMB,SPI,FC,BRATE,DELT,TO,TB,ISW,V,THETA,FFCTR,CALSQ,
     1 VW,VCW,ALT,R,IEND,CMACH,REYNLO,RESIS,CAL,DLONG,IOPTY,YAW,AMOM,
     2 BMOM,PSI,WTAREA,ISEP,NABLE,IGLIDE
      IF(TIME.LE.TO) GO TO 1
      IF(TIME.LE.TO+OELT) GO TO 2
      T2=TO+TB
      IF(TIME.LE.T2) GO TO 3
      IF(TIME.LT.T2+OELT) GO TO 4
      THRUST=0.
      XMASS=EMB
      RETURN
```

```
      1 XMASS=EMO
        THRUST=0.
        RETURN
      2 XMASS=EMO-BRATE*(TIME-TO)**2/(2.*DELT)
        THRUST=(TIME-TO)/DELT*FC
        RETURN
      3 XMASS=EMO-BRATE*(TIME-TO-DELT/2.)
        THRUST=FC
        RETURN
      4 XMASS=EMO-BRATE*((TB-DELT/2.)+(TIME-T2)*(1.-(TIME-T2)/DELT/2.))
        THRUST=FC*(1.-(TIME-T2)/DELT)
        RETURN
        END
        FUNCTION IBURN(TIME,ALT,QE,VELO)
C
C       FUNCTION PRODUCES INDICATION OF COMMENCEMENT OF
C       BURNING--IBURN=1.
C       IBURN=0 UNTIL BURN BEGINS
C       USER CHOOSES TO FROM CURRENT TIME OR ALT (ALTITUDE) OR
C       QE (LOCAL QUADRANT ELEVATION) OR VELO (VELOCITY)
        COMMON /SRCOM/TM
        DATA ALTMAX/30000./,VMIN/0.0/
        IF(TIME.GE.TM) GO TO 3
        IF(ALT.GE.ALTMAX) GO TO 3
        IF(VELO.LE.VMIN) GO TO 3
C       IF(QE.GT.45.0) GO TO 2
        IBURN=0
        RETURN
C     2 IF(QE.LT.46.0) GO TO 3
C     4 IBURN=0
C       RETURN
      3 IBURN=1
        RETURN
        END

IN NM DIRECTORY. TTR IS NOW ALTERED.
0000000
```

AERO DATA (BRL CALC) FOR M509 8 IN ICM PROJECTILE

MACH NO	COEF DRAG	DRAG INCR
0.0	0.1300	0.0
0.7500	0.1300	0.0
0.8500	0.1400	0.0
0.9000	0.1550	0.0
1.0000	0.3000	0.0
1.0500	0.3600	0.0
1.1000	0.3600	0.0
1.5000	0.3170	0.0
2.0000	0.2740	0.0
2.5000	0.2390	0.0

ENABLE TIME = 100.0000 THRUST DRAG FACTOR = 1.0000 AIR DENS FACTOR = 1.0000

```
EXTERIOR BALLISTICS OF THE M509 (WITH XM42 SUBMSL) & VARIATIONS

        FFCTR            VO             MO             MB             D
     1.000000    1040.00000     205.899994     205.899994     203.199997

    QUAD ELEV        TM STEP         THRUST    SP IMPULSE         V-WIND
   12.000000       0.100000       0.0           0.0            0.0

  INIT ALT,FT    TERM ALT,FT VEL XWIND,FT/S            RWC            XWC
     0.0           0.0            0.0           0.0            0.0

VHXF =          0.0  FT/SEC      VHYF =          0.0  FT/SEC

THRUST RISE TIME =        0.0     SEC   TM =     100.0000  SEC

LOC OF CG =0.3577E+01  CAL PROJ LENGTH =0.5673E+01  CAL AXIAL M OF I =0.54700E+00  KG M**2 TRANS M OF I =0.47676E+01  KG M**2
```

EXTERIOR BALLISTICS OF THE M509 (WITH XM42 SUBMSL) & VARIATIONS

TIME,SECS	X,M	ALT,M	Z,M	XDOT,M/S	YDOT,M/S	V,M/S	MACH NO.	DRAG,LB	THETA,D	YAW,D	SPIN,R/S	STA FAC	DUMMY V
0.0	0.0	0.0	0.0	310.1	65.9	317.0	0.93	-89.9	12.0	0.0	490.00	1.93	6.20
1.000	308.1	60.6	0.1	306.2	55.4	311.2	0.91	-75.9	10.2	0.26	486.81	2.01	6.30
2.000	612.6	110.8	0.2	302.9	45.0	306.2	0.90	-64.7	8.5	0.27	483.69	2.09	6.38
3.000	914.0	150.8	0.6	299.9	34.9	302.0	0.89	-60.9	6.6	0.29	480.63	2.14	6.40
4.000	1212.5	180.7	1.0	297.1	24.8	298.2	0.88	-57.9	4.8	0.30	477.62	2.18	6.41
5.000	1508.3	200.6	1.6	294.4	14.9	294.8	0.87	-55.4	2.9	0.31	474.66	2.21	6.41
6.000	1801.4	210.6	2.2	291.8	5.0	291.9	0.86	-53.3	1.0	0.32	471.74	2.23	6.40
7.000	2092.0	210.8	3.1	289.3	-4.8	289.4	0.85	-51.6	-0.9	0.32	468.86	2.24	6.37
8.000	2380.1	201.2	4.0	286.9	-14.5	287.3	0.85	-50.5	-2.9	0.33	466.00	2.25	6.34
9.000	2665.8	182.0	5.1	284.5	-24.1	285.5	0.84	-49.8	-4.8	0.33	463.17	2.25	6.30
10.000	2949.1	153.3	6.3	282.2	-33.6	284.2	0.84	-49.3	-6.8	0.33	460.36	2.24	6.25
11.000	3230.1	115.1	7.7	279.8	-43.1	283.1	0.83	-49.0	-8.8	0.33	457.56	2.22	6.19
12.000	3508.8	67.4	9.2	277.5	-52.5	282.5	0.83	-48.9	-10.7	0.33	454.77	2.19	6.12
13.000	3785.2	10.4	10.8	275.3	-61.8	282.1	0.83	-49.0	-12.7	0.32	451.99	2.16	6.05
13.133	3821.9	0.0	11.0	274.9	-63.0	282.1	0.83	-49.0	-12.9	0.32	451.62	2.16	3821.91

MAX PROJ VELOCITY = 1039.9998 F/S MAX RANGE = 2.0637 NAUT MILES MAX ALT = 211.8979 METERS

EXTERIOR BALLISTICS OF THE M509 (WITH XM42 SUBMSL) & VARIATIONS

QE= 12.0 DEG, V0=1040.0 F/S

Computer Program for the Gyroscopic Dynamics of Projectiles Having Oscillating Inertial Properties

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FORTRAN IV G LEVEL  21                MAIN                DATE = 75301          13/47/12              PAGE 0001

              C     GYRO DYNAMICS
              C**** THIS PROGRAM PRODUCES A NUMERICAL INTEGRATION OF THE
              C**** DIFFERENTIAL EQUATIONS DESCRIBING THE DYNAMICS OF A SPIN-
              C**** STABILIZED PROJECTILE HAVING OSCILLATING CENTER OF PRESSURE
              C**** AND TRANSVERSE MOMENT OF INERTIA.  THE PROGRAM EMPLOYS A
              C**** DOUBLE-PRECISION VERSION OF THE FOURTH-ORDER RUNGE-KUTTA
              C**** METHOD IN SUBROUTINE DKUTTA.  THE DIFFERENTIAL EQUATIONS
              C**** ARE LOCATED IN SUBROUTINE DIFEQ.
              C*****
0001                IMPLICIT REAL *8 (A-H,O-Z)
0002                DIMENSION TITLE(20),U(8),WP(32),X(4,2)
0003                COMMON CALIB,CNA,CMQ,CD,OCG,OJ,DCG,DJ,C31,C32,C33,C34,CP,
                  1 OMEGA,DOMEGA,PMASS
              C**** READ TITLE AND INPUT PARAMS
              C****
              C**** INPUT VARIABLES:
              C**** TITLE = ALPHAMERIC DESCRIPTION OF SYSTEM BEING STUDIED
              C**** DT = INTEGRATION TIME STEP (SEC)
              C**** TMAX = MAXIMUM TIME PERMITTED FOR SOLUTION (SEC)
              C**** THETAO = INITIAL VALUE OF PITCH (RAD)
              C**** PSIO = INITIAL VALUE OF YAW (RAD)
              C**** THEDO = INITIAL VALUE OF PITCH RATE (RAD/SEC)
              C**** PSIDO = INITIAL VALUE OF YAW RATE (RAD/SEC)
              C**** TORQUE = EXTERNAL TORQUE APPLIED IN PITCH (DYNE CM)
              C**** NPRINT = NUMBER OF TIME STEPS BETWEEN PRINTOUTS
              C**** ISW = SWITCH FOR OPTION NOT EXERCISED IN PRESENT VERSION
              C**** FREQ = THE CONSTANT PART OF THE FORCING FREQUENCY (HZ)
              C**** DFREQ = THE SWEEP RATE OR FIRST TIME-DERIVATIVE OF THE
              C**** FORCING FREQUENCY (HZ/SEC)
              C**** TS = SAMPLING INTERVAL USED IN THE SEMIANALYTIC MODEL (SEC)
              C**** SEE NOMENCLATURE BELOW FOR PROJECTILE PARAMETERS.
              C**** ALL DIMENSIONS MUST BE IN A RATIONALIZED SET OF UNITS SUCH AS:
              C**** LENGTH IN FT, MASS IN SLUGS, FORCE IN POUNDS, AND TIME IN SEC.
0004                READ (5,100) TITLE,DT,TMAX,THETAO,PSIO,THEDO,PSIDO,TORQUE,NPRINT,
                  1 ISW
0005            100 FORMAT (20A4/7F10.0,2I2)
0006                READ (5,110) FREQ,DFREQ,TS
0007            110 FORMAT(3F10.0)
0008                WRITE (6,200) TITLE,DT,TORQUE
0009            200 FORMAT (1H1,20A4/1H0,11HTIME STEP =,E15.5,2X,3HSEC,10H  TORQUE =,
                  1 E15.5,2X,7HDYNE CM)
0010                WRITE (6,115) FREQ,DFREQ,TS
0011            115 FORMAT(1H ,5X,7HFREQ = ,F10.2,6H  HZ  ,7HDFREQ = ,F10.2,6HHZ/S  ,
                  1 10HSAMP INT =,F10.3,2X,3HSEC)
0012            400 FORMAT(1H ,6F15.6)
0013            121 FORMAT(8F10.0)
0014                READ (5,121) CALIB,PMASS,AXMI,OJ,DJ,VO,SPIN,RHO,
                  1 CD,CNA,CMQ,CMPA,CP,OCG,DCG
0015                WRITE (6,122) CALIB,PMASS,AXMI,OJ,DJ,VO,SPIN,RHO,
                  1 CD,CNA,CMQ,CMPA,CP,OCG,DCG
0016            122 FORMAT(1H0,5X,5HCALIB,5X,5HPMASS,5X,5HAX MI,2X,8HAVG T MI,2X,
                  1 8HINC T MI,8X,2HVO,6X,4HSPIN,7X,3HRHO/7F10.4,1F10.6/
                  2 8X,2HCD,7X,3HCNA,7X,3HCMQ,6X,4HCMPA,8X,2HCP,4X,6HAVG CG,
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                3 4X,6HINC CG/7FI0.4)
          C**** COMPUTE CONSTANTS IN OIFFERENTIAL EQUATIONS
 0017           AREA=0.7853982*CALIB**2
 0018           OYNPRS=0.5*RHO*VO**2
 0019           C31=AREA*DYNPRS*CALIB
 0020           GYRXS=AXMI/PMASS/CALIB/CALIB
 0021           C32=-SPIN/VO*CALIB*GYRXS*C3I*(CNA-CO+CMPA/GYRXS)
 0022           C33=-AREA*DYNPRS/VO/PMASS
 0023           C34=-AXMI*SPIN
 0024           CMA=CNA*(OCG-CP)
 0025           STAFAC=C34**2/4./OJ/C31/CMA
          C**** THE HIGH- AND LOW OSCILLATORY MOOES OF THE PROJECTILE ARE
          C**** TERMEO FNUI ANO FPREC, RESPECTIVELY.
          C**** THE FREQUENCY WITH WHICH YAW MAXIMA OCCUR IS OENOTEO FOIF.
 0026           SIGMA=DSQRT(I.-I./STAFAC)
 0027           SAVE=0.0795775*AXMI/OJ*SPIN
 0028           FNUT=SAVE*(I.+SIGMA)
 0029           FPREC=SAVE*(I.-SIGMA)
 0030           FOIF=FNUT-FPREC
 0031           WRITE (6,124) STAFAC,FNUT,FPREC,FDIF
 0032       124 FORMAT(1H0,I5HNOM STAB FAC = ,FI0.3/3X,
               I 13HNUTAT FREQ = ,FI0.3,17H HZ  PREC FREQ = ,F10.3,
               2 16H HZ DIFF FREQ = ,FI0.3,3H HZ)
          C**** NOMENCLATURE:
          C**** CALIB IS REFERENCE OIAMETER OF THE PROJECTILE
          C**** AREA IS THE PROJECTILE REFERENCE AREA
          C**** PMASS IS THE MASS OF THE PROJECTILE
          C**** AXMI IS THE AXIAL MOMENT OF INERTIA
          C**** OJ  IS THE NOMINAL TRANSVERSE MOMENT OF INERTIA
          C**** DJ IS THE AMPLITUDE OF THE EXCURSION IN THE TRANS. MOMENT OF INERTIA
          C**** VO IS THE MUZZLE VELOCITY OF THE PROJECTILE
          C**** SPIN IS THE PROJECTILE SPIN IN RAO/SEC
          C**** RHO IS THE AIR DENSITY.
          C**** CD IS THE DRAG COEFFICIENT
          C**** CNA IS THE NORMAL FORCE OERIVATIVE COEFFICIENT (PER RAO)
          C**** CMO IS THE COMPOSITE OAMPING COEFFICIENT (PER RAD/SEC)
          C**** CMPA IS THE MAGNUS MOMENT DERIVATIVE COEFFICIENT
          C**** CP IS THE CENTER OF PRESSURE AFT OF THE NOSE IN CALIBERS
          C**** CG IS THE POSITION OF THE CENTER OF GRAVITY AFT OF THE NOSE (CAL)
          C**** OCG IS THE AVERAGE POSITION OF THE CG
          C**** OCG IS THE AMPLITUDE OF THE EXCURSION OF THE CG (CAL)
          C**** PRINT COLUMN HEAOINGS
 0033       300 FORMAT (1H0,I1X4HTIME,I0X5HPITCH,I2X3HYAW,5X10HPITCH RATE,7X,
               I 8HYAW RATE,5X,I0HRSSQ ANGLE,11X,4HFREQ)
 0034           WRITE (6,300)
 0035           OMEGA=6.2832*FREQ
 0036           OOMEGA=6.2832*DFREQ
 0037           TAMAX=0.0
 0038           AMAX=0.0
          C**** INITIALIZE STATE VECTOR
 0039           U(1)=THETAO
 0040           U(2)=PSIO
 0041           U(3)=THEDO
```

```
0042              U(4)=PSIDO
            C**** INITIALIZE RUNGE-KUTTA SUBROUTINE
            C**** SOLVE DIFF EQNS FOR DERIVATIVE ICS
0043              T=0.
0044              CALL DIFEQ(T,U,4)
0045           19 KOUNT=0
            C
            C**** START OF SOLUTION LOOP
0046           20 CONTINUE
            C**** MOVE STATE FROM T TO T+DT
0047              CALL DKUTTA(T,DT,U,WP,4,2,DIFEQ)
0048              SAVE=U(1)**2+U(2)**2
0049              IF(SAVE.GT.AMAX) GO TO 22
0050           21 CONTINUE
0051              IF(T.GT.TMAX) GO TO 42
0052              KOUNT=KOUNT+1
0053              IF(KOUNT.EQ.NPRINT) GO TO 40
0054              GO TO 20
0055           22 AMAX=SAVE
0056              TAMAX=T
0057              GO TO 21
0058           40 TP=T
0059              ATHET=57.2958*U(1)
0060              APSI=57.2958*U(2)
0061              DATHET=57.2958*U(3)
0062              DAPSI=57.2958*U(4)
0063              RSSX=DSQRT(U(1)**2+U(2)**2) *57.2958
0064              FFREQ=FREQ+T*DFREQ
            C     AWRTDV=57.2958*DATAN2(-DSIN(U(2))*DCOS(U(1)),DSIN(U(1))*
            C    1 DCOS(U(2)))
0065              WRITE (6,900) TP,ATHET,APSI,DATHET,DAPSI,RSSX,FFREQ
0066          900 FORMAT(1H ,7F15.6)
0067              IF(RSSX.GE.20.00) CALL EXIT
0068              GO TO 19
0069           42 AMAX=57.2958*DSQRT(AMAX)
0070              WRITE (6,44) TAMAX,AMAX
0071           44 FORMAT(1H0,3HAT ,F12.6,20H SEC THE MAX YAW OF ,F12.6,
                 1 11H DEG EXISTS)
0072              CALL EXIT
0073              END
```

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FORTRAN IV G LEVEL  21                DIFEQ               DATE = 75301          13/47/12              PAGE 0001

0001               SUBROUTINE DIFEQ(TIME,U,KUTTA)
             C**** DIFFERENTIAL EQUATIONS FOR A SPIN-STABILIZED PROJECTILE
0002               IMPLICIT REAL * 8 (A-H,D-Z)
0003               DIMENSION U(8)
0004               COMMON CALIB,CNA,CMQ,CD,OCG,OJ,DCG,DJ,C31,C32,C33,C34,CP,
                  1 OMEGA,DOMEGA,PMASS
             C**** U(1) IS THE PITCH ANGLE AND U(2) IS THE YAW ANGLE
0005               SINWT=DSIN((OMEGA+DOMEGA*TIME)*TIME)
0006               XJ=OJ+DJ*SINWT
0007               CG=OCG+DCG*SINWT
0008               CMA=CNA*(CG-CP)
0009               A31=C31*CMA/XJ
0010               GYRYS=XJ/PMASS/CALIB/CALIB
0011               A32=C32/XJ
0012               A33=C33*(CNA-2.*CD-CMQ/GYRYS)
0013               A34=C34/XJ
0014               A41=-A32
0015               A42=A31
0016               A43=-A34
0017               A44=A33
0018               U(5)=U(3)
0019               U(6)=U(4)
0020               U(7)=A31*U(1)+A32*U(2)+A33*U(3)+A34*U(4)
0021               U(8)=A41*U(1)+A42*U(2)+A43*U(3) +A44*U(4)
0022               RETURN
0023               END
```

STABILITY ANALYSIS OF THE XM410 PROJECTILE AT MACH 1.5

TIME STEP = 0.20000D-03 SEC TORQUE = 0.0 DYNE CM
FREQ = 17.50 HZ DFREQ = 0.0 HZ/S SAMP INT = 0.0 SEC

CALIB	PMASS	AX MI	AVG T MI	INC T MI	VO	SPIN	RHO
0.5000	1.3130	0.0446	0.1548	0.0055	1675.5000	526.4000	0.002377
CD	CNA	CMQ	CMPA	CP	AVG CG	INC CG	
0.5000	2.9000	-5.0000	0.3000	1.4000	1.8500	0.0300	

NOM STAB FAC = 2.083
NUTAT FREQ = 20.770 HZ PREC FREQ = 3.367 HZ DIFF FREQ = 17.403 HZ

TIME	PITCH	YAW	PITCH RATE	YAW RATE	RSSQ ANGLE	FREQ
0.010000	1.192863	1.356268	-1.027796	42.675685	1.806208	17.500000
0.020000	0.957394	1.890347	-50.060366	54.234828	2.118966	17.500000
0.030000	0.239986	2.228883	-84.614976	5.339857	2.241766	17.500000
0.040000	-0.492864	1.973189	-50.700330	-49.380244	2.033812	17.500000
0.050000	-0.694811	1.489465	5.058644	-34.613641	1.643554	17.500000
0.060000	-0.618095	1.401928	-2.361505	14.591277	1.532137	17.500000
0.070000	-0.897323	1.603213	-54.294940	13.707221	1.837248	17.500000
0.080000	-1.581653	1.477499	-71.123824	-43.925565	2.164401	17.500000
0.090000	-2.069888	0.774237	-16.892781	-86.567611	2.209951	17.500000
0.100000	-1.893577	0.025862	43.715615	-50.411481	1.893754	17.500000
0.110000	-1.477902	-0.152208	25.449952	8.592103	1.485719	17.500000
0.120000	-1.512374	-0.052227	-29.483298	-2.287692	1.513275	17.500000
0.130000	-1.873062	-0.368487	-29.432447	-62.118844	1.908964	17.500000
0.140000	-1.874365	-1.163676	35.308607	-84.166517	2.206215	17.500000
0.150000	-1.217303	-1.768965	84.686855	-26.050138	2.147339	17.500000
0.160000	-0.492184	-1.660700	46.622776	38.017171	1.732100	17.500000
0.170000	-0.365501	-1.316810	-14.336825	16.035492	1.366594	17.500000
0.180000	-0.513031	-1.471696	-0.713600	-43.972127	1.558554	17.500000
0.190000	-0.194805	-1.990007	65.987096	-45.376288	1.999519	17.500000
0.200000	0.670290	-2.126323	93.124219	25.236950	2.229470	17.500000
0.210000	1.356254	-1.537915	32.325100	79.816929	2.050514	17.500000
0.220000	1.298367	-0.867651	-33.122251	39.893667	1.561594	17.500000
0.230000	1.019907	-0.817269	-7.435601	-22.347273	1.306958	17.500000
0.240000	1.284240	-1.038363	57.245424	-6.797090	1.651506	17.500000
0.250000	1.950134	-0.753341	60.628556	66.003093	2.090585	17.500000
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0.270000	1.712551	0.869971	-72.828548	35.436722	1.920854	17.500000
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0.290000	1.167075	0.611037	32.417425	-0.652554	1.317358	17.500000
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0.310000	1.266275	1.759634	-62.551215	74.255431	2.167894	17.500000
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0.330000	-0.354460	1.728425	-35.299549	-64.616829	1.764397	17.500000
0.340000	-0.326088	1.229501	28.682381	-20.518778	1.272009	17.500000
0.350000	-0.122935	1.386720	-3.464059	44.057694	1.392158	17.500000
0.360000	-0.538266	1.823246	-76.938666	27.000426	1.901041	17.500000
0.370000	-1.435763	1.694084	-85.414628	-56.270775	2.220661	17.500000
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0.430000	-2.002077	-1.006783	47.990996	-93.451212	2.240965	17.500000
0.440000	-1.220098	-1.570842	90.420416	-10.165361	1.989016	17.500000
0.450000	-0.576637	-1.298023	25.867896	48.202006	1.420343	17.500000
0.460000	-0.685194	-0.989116	-34.769166	-0.647269	1.203263	17.500000

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